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معامل التأثير العربي 1.5
العدد 20

مجلة التربوي
مجلة علمية محكمة تصدر عن كلية التربية
جامعة المرقب

العدد العشرون
يناير 2022م

هيئـــة تحريـر
مجلة التربوي

- المجلة ترحب بما يرد عليها من أبحاث وعلى استعداد لنشرها بعد التحكيم .
- المجلة تحترم كل الاحترام آراء المحكمين وتعمل بمقتضاها .
- كافة الآراء والأفكار المنشورة تعبر عن آراء أصحابها ولا تتحمل المجلة تبعاتها .
- يتحمل الباحث مسؤولية الأمانة العلمية وهو المسؤول عما ينشر له .
- البحوث المقدمة للنشر لا ترد لأصحابها نشرت أو لم تنشر .

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ضوابط النشر:

يشترط في البحوث العلمية المقدمة للنشر أن يراعى فيها ما يأتي :

- أصول البحث العلمي وقواعده .
- ألا تكون المادة العلمية قد سبق نشرها أو كانت جزءا من رسالة علمية .
- يرفق بالبحث تزكية لغوية وفق أنموذج معد .
- تعدل البحوث المقبولة وتصحح وفق ما يراه المحكمون .
- التزام الباحث بالضوابط التي وضعتها المجلة من عدد الصفحات ، ونوع الخط ورقمه ، والفترات الزمنية الممنوحة للتعديل ، وما يستجد من ضوابط تضعها المجلة مستقبلا .

تنبيهات :

- للمجلة الحق في تعديل البحث أو طلب تعديله أو رفضه .
- يخضع البحث في النشر لأولويات المجلة وسياستها .
- البحوث المنشورة تعبر عن وجهة نظر أصحابها ، ولا تعبر عن وجهة نظر المجلة .

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Oscillation Criterion for Second Order Nonlinear Differential Equations

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Abstract:

In this paper, some the sufficient conditions for the oscillation of the solutions of the second order non-linear ordinary differential equation of the form

$$\left(r(t)\psi(x)\dot{x}(t)\right)^{\bullet} + q(t)\Phi(g(x(t)), r(t)\dot{x}(t)) = H(t, x(t))$$

are obtained using Riccati Technique. The given results are the extension and improvement of the results of oscillation which were obtained before by many authors as Bihari [2] and Kartsatos [7]. These results are illustrated with examples that are solved using Runge Kutta method of forth order.

1. Introduction

Consider the second order non-linear ordinary differential 2equation of the form

$$\left(r(t)\psi(x)\dot{x}(t)\right)^{\bullet} + q(t)\Phi(g(x(t)), r(t)\dot{x}(t)) = H(t, x(t)) \qquad (E)$$

where r, ψ and q are continuous functions on the interval $[t_0, \infty)$, $t_0 \geq 0$, $r(t)$ is a positive function, g is continuously differentiable function on the real line R except possibly at 0 with $xg(x) > 0$ and $g'(x) \geq k > 0$ for all $x \neq 0$, Φ is a continuous function on RxR with $u\Phi(u,v) > 0$ for all $u \neq 0$ and $\Phi(\lambda u, \lambda v) = \lambda\Phi(u,v)$ for any $(\lambda, u, v) \in \mathrm{R}^3$ and H is a continuous function on $[t_0, \infty) \times \mathrm{R}$ with $H(t, x(t))/g(x(t)) \leq p(t)$ for all $x \neq 0$ and $t \geq t_0$.Throughout this study, we restrict our attention only to the solutions of the differential ordinary equation (E) that exist on some ray $[t_x, \infty)$, where t_x may depend on the particular solution. A solution $x(t)$ of the differential equation (E) is said to be oscillatory if it has arbitrary large zeros, and otherwise it is said to be.non-oscillatory. Equation (E) is called oscillatory if all its solutions are oscillatory, and otherwise it is called non oscillatory. Particular cases of the equation (E) have been considered by many authors for example [1-13]. Some of these particular cases can be classified as follows

$$\ddot{x}(t) + q(t)x(t) = 0 \qquad (1)$$

$$\ddot{x}(t) + q(t)\,\Phi(x(t), \dot{x}(t)) = 0 \qquad (2)$$

$$\left(r(t)\,\dot{x}(t)\right)^{\bullet} + q(t)\,g(x(t)) = H\left(t, x(t)\right) \qquad (3)$$

The oscillation of linear equation (1) has brought the attention of many authors since because of Fite [3]. He proved that if $q(t) > 0$ for all $t \geq t_0$ and $\int_{t_0}^{\infty} q(s)\, ds = \infty$, then every solution of the equation (1) is oscillatory. Wintner [12] extended the result of Fite [3] to an equation in which q is of arbitrary sign and supposed that

$$\lim_{t\to\infty} \frac{1}{t} \int_{t_0}^{t} (t-s)\, q(s)\, ds = \infty,$$

then, every solution of the equation (1) is oscillatory. In the following, Kamenev [6] has proved a new integral criterion for the oscillation of the differential equation (1) based on the use of the n the primitive of the coefficient $q(t)$, which has Wintner's result [12] as a particular case. He has showed that the equation (1) is oscillatory if

$$\lim_{t\to\infty} \sup \frac{1}{t^{n-1}} \int_{t_0}^{t} (t-s)^{n-1} q(s)\, ds = \infty,$$

for some integer $n \geq 3$. The oscillation of the equation (2) has brought the attention of some authors because of Bihari [2], who has proved that if $q(t) > 0$ *for all* $t \geq t_0$ and

$$\int_{t_0}^{\infty} q(s)ds = \infty,$$

then, every solution of the equation (2) is oscillatory. The following result extended the result of Bihari [2] to an equation in which q is of arbitrary sign, in this theorem, Kartsatos [7] has supposed

(i) There exists a constant $C \in R_- = (-\infty, 0)$ such that

$$G(m) = \int_0^u \frac{du}{\Phi(1,u)} \geq -C \text{ for all } u \in R,$$

(ii) $\int_{t_0}^{\infty} q(s)ds = \infty.$

Then, every solution of equation (2) is oscillatory. Many authors are concerned with the oscillation criteria of solutions of the homogeneous second order nonlinear differential equations. However, of the non-homogeneous equation, little is known. Greaf, Rankin and Spikes [5] gave some theorems for the non-homogeneous equation (3) for example, they proved that if

(1) $r(t) \leq a_1, a_1 > 0,$

(2) $\lim_{t \to \infty} \frac{1}{t} \int_{t_0}^{t} \int_{t_0}^{s} \left(q(u) - p(u)\right) du ds = \infty,$

then, all solutions of equation (3) are oscillatory.

2. MAIN RESULTS

In this section, Riccati technique is used to reduce the higher-order equations to the first-order Riccati equation or inequality to establish sufficient conditions for oscillation of (E). Comparisons between our results and the previously known are presented and some examples illustrate the main results.

Theorem2.1: Suppose that

$$(1)\ b_1 \le \psi(x) \le b_2,\ \ b_1, b_2 > 0 \text{ for all } x \in IR.$$

$$(2)\qquad \liminf_{v\to\infty} \frac{1}{\Phi(1,v)} \ge C_0,\ C_0 > 0.,$$

$$(3)\qquad G(m) = \int_0^m \frac{ds}{\Phi(1,s)} > -B^*, B^* > 0\ \ for\ \ every\, m \in \mathrm{R}^+.$$

Assume that there exists ρ be a positive continuous differentiable function on the interval $[t_0,\infty)$ with $\rho(t)$ is increasing on the interval $[t_0,\infty)$ and such that

$$(4)\qquad \limsup_{t\to\infty} \frac{1}{\rho(t)} \int_T^t \rho(s)[C_0 q(s) - p(s)]ds = \infty,$$

where, $p:[t_0,\infty) \to (0,\infty)$, then every solution of equation (E) is oscillatory.

Proof:

Without loss of generality, we may assume that there exists a solution $x(t)$ of equation (E) such that $x(t) > 0\ on\ [T,\infty)\ \ for\ \ some\ T \ge t_0 \ge 0.$ Define

$$\omega(t) = \frac{\rho(t) r(t) \psi(t) \dot{x}(t)}{g(x(t))},\ t \ge T$$

Thus by condition (1) and equation (E) imply

$$\dot{\omega}(t) \le \rho(t)p(t) - \rho(t)q(t)\Phi(1,\omega(t)/\rho(t)) + \frac{\dot{\rho}(t)}{\rho(t)}\omega(t) - \frac{k}{b_2\rho(t)r(t)}\omega^2(t),\ t \ge T$$

Thus, we have

$$\rho(t)\left(\frac{\omega(t)}{\rho(t)}\right)^{\bullet} \le \rho(t)p(t) - \rho(t)q(t)\Phi(1,\omega(t)/\rho(t)) - \frac{k}{b_2\rho(t)r(t)}\omega^2(t),\ t \ge T \qquad (2\text{-}1)$$

Dividing the last inequality by $\Phi(1,\omega(t)/\rho(t)) > 0$, we have

$$\frac{\rho(t)(\omega(t)/\rho(t))^{\bullet}}{\Phi(1,\omega(t)/\rho(t))} \le \frac{\rho(t)p(t)}{\Phi(1,\omega(t)/\rho(t))} - \rho(t)q(t),\ t \ge T$$

By condition (2),we find $\Phi(1,\omega(t)/\rho(t)) \ge C_0$, then for $t \ge T$, we obtain

$$\rho(t)[C_0q(t) - p(t)] \le -\frac{C_0\rho(t)(\omega(t)/\rho(t))^{\bullet}}{\Phi(1,\omega(t)/\rho(t))},\ t \ge T$$

Integrate the last inequality from T to t, we obtain

$$\int_T^t \rho(s)[C_0q(s) - p(s)]ds \le -C_0\int_T^t \frac{\rho(s)(\omega(s)/\rho(s))^{\bullet}}{\Phi(1,\omega(s)/\rho(s))}ds\ ,t \ge T \qquad (2\text{-}2)$$

By the Bonnet's theorem, we see that for each $t \ge T$, there exists $T_1 \in [T,t]$ such that

$$-\int_T^t \frac{\rho(s)(\omega(s)/\rho(s))^{\bullet}}{\Phi(1,\omega(s)/\rho(s))}ds = -\rho(t)\int_{T_1}^t \frac{(\omega(s)/\rho(s))^{\bullet}}{\Phi(1,\omega(s)/\rho(s))}ds \qquad (2\text{-}3)$$

From inequality (2-3) in inequality (2-2), we have

$$\int_T^t \rho(s)[C_0 q(s) - p(s)]ds \le -C_0 \rho(t) \int_{T_1}^t \frac{(\omega(s)/\rho(s))^{\bullet}}{\Phi(1,\omega(s)/\rho(s))} ds = -C_0 \rho(t) \int_{\omega(T_1)/\rho(T_1)}^{\omega(t)/\rho(t)} \frac{du}{\Phi(1,u)}$$

By condition (3), dividing the last inequality by $\rho(t)$ and taking the limit superior on both sides, we obtain

$$\begin{aligned} \limsup_{t\to\infty} \frac{1}{\rho(t)} \int_T^t \rho(s)[C_0 q(s) - p(s)]ds &\le -C_0 \limsup_{t\to\infty} \int_{\omega(T_1)/\rho(T_1)}^{\omega(t)/\rho(t)} \frac{du}{\Phi(1,u)} \\ &\le -C_0 \limsup_{t\to\infty} \left[- \int_0^{\omega(T_1)/\rho(T_1)} \frac{du}{\Phi(1,u)} + \int_0^{\omega(t)/\rho(t)} \frac{du}{\Phi(1,u)} \right] \\ &\le C_0 \limsup_{t\to\infty} \left(G\left(\frac{\omega(T_1)}{\rho(T_1))} \right) + B^* \right) < \infty, \end{aligned}$$

as $t \to \infty$, which contradicts to the condition (4). Hence the proof is completed.

Example2.1

Consider the differential equation

$$\left(t \, \frac{x^2(t)+4}{x^2(t)+3} \, \dot{x}(t) \right)^{\bullet} + \left(\frac{t^3 + 3\cos t}{t^2} \right) x(t) = \frac{x(t)\cos x(t)}{t^4}, \; t > 0$$

Here $r(t) = t$, $\psi(t) = \dfrac{x^2(t)+4}{x^2(t)+3}$ $q(t) = \dfrac{t^3 + 3\cos t}{t^2}$, $g(x) = x$, $\Phi(u,v) = u$ and $\dfrac{H(t,x(t))}{g(x(t))} = \dfrac{\cos x(t)}{t^4} \le \dfrac{1}{t^4} = p(t)$ *for all* $t > 0$ *and* $x \ne 0$. Taking $\rho(t) = t^2$ *such that*

$$\lim_{t\to\infty}\frac{1}{\rho(t)}\int_T^t \rho(s)\big(C_0q(s)-p(s)\big)ds=\lim_{t\to\infty}\frac{1}{t^2}\int_T^t s^2\left(\frac{C_0s^3+3C_0\cos s}{s^2}-\frac{1}{s^4}\right)ds=\infty.$$

All conditions of theorem2.1 are satisfied and hence every solution of the given equation is oscillatory. To ensure that our result in theorem2.1 is true we also find the numerical solutions of the given differential equation in example 2.1 using the Runge Kutta method of fourth order (RK4). We have

$$\ddot{x}(t)= f(t,x(t),\dot{x}(t))=x\cos(x)-3.99x$$

with initial conditions $x(1)=1,\dot{x}(1)=-0.5$ on the chosen interval $[1,100]$, the function $\psi\equiv 1$, and finding values of the functions r, q and f where we consider $H(t,x(t))= f(t)l(x)$ at $t=1$, $n=500$ and $h=0.198$

K	t_k	$x(t_k)$
1	1	1
2	1.1980	0.8370
3	1.3960	0.5662
4	1.5940	0.2258
5	1.7920	-0.1412
6	1.9900	-0.4917
7	2.1880	-0.7824
8	2.3860	-0.9733
9	2.5840	-1.0352
10	2.7820	-0.9578
11	2.9800	-0.7539
12	3.1780	-0.4544
13	3.3760	-0.0999
14	3.5740	0.2663
15	3.7720	0.6009
16	3.9700	0.8617

Table 1: Numerical solution of ODE1

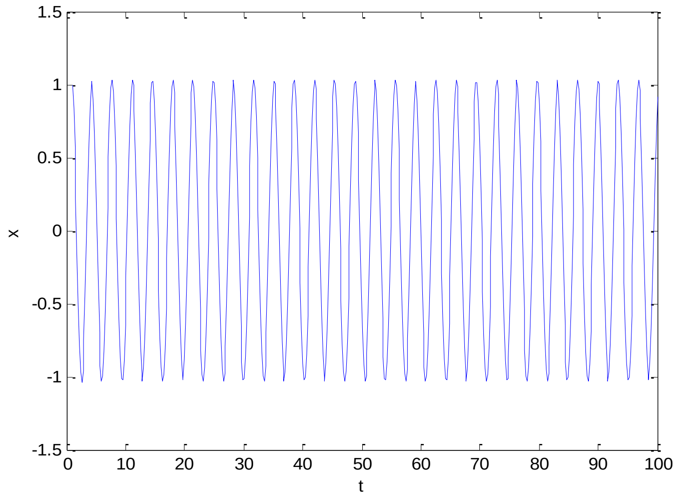

Figure1: Solution curve of ODE 1

Remark 2.1: Theorem 2.1 is extension of the results of Bihari [2], Kartsatos [7], Kamenev [6] and Wintiner [12]. All results of them [2], [7], [6] and [12] cannot be applied to the given equation in example2.1.

Theorem 2.2: Suppose, in addition to the condition (1) that

$$(5) \qquad \limsup_{v\to\infty} \frac{1}{\Phi(1,v)} \le C_1,\ C_1 > 0.$$

$$(6) \qquad \frac{1}{\Phi(1,v)} \le v \ \ \textit{for all } v \ne 0.$$

Assume that ρ be a positive continuous differentiable function on the interval $[t_0,\infty)$ with $\rho(t)$ is a creasing function on the interval $[t_0,\infty)$ and such that

$$(7) \qquad \limsup_{t\to\infty} \int_T^t \rho(s)\left[q(s) - \frac{1}{4k^*} p^2(s)\right] ds = \infty,$$

where, $p:[t_0,\infty)\to(0,\infty)$, then every solution of equation (E) is oscillatory.

Proof: Without loss of generality, we may assume that there exists a solution $x(t)$ of equation (E) such that $x(t) > 0$ *on* $[T,\infty)$ *for some* $T \geq t_0 \geq 0$. By conditions (5) and (6) and from inequality (2-1) divided by $\Phi(1,\omega(t)/\rho(t)) > 0$, we have

$$\frac{\rho(t)(\omega(t)/\rho(t))^{\bullet}}{\Phi(1,\omega(t)/\rho(t))} \leq p(t)\omega(t) - \rho(t)q(t) - \frac{k^*}{r(t)\rho(t)}\omega^2(t)$$

where $k^* = k/b_2C_1$.

Integrate the last inequality from T to t, we obtain

$$\int_T^t \frac{\rho(s)(\omega(s)/\rho(s))^{\bullet}}{\Phi(1,\omega(s)/\rho(s))} ds \leq -\int_T^t \rho(s)q(s)ds - \int_T^t \left[\frac{k^*}{r(s)\rho(s)}\omega^2(t) - p(s)\omega(s)\right] ds$$

Thus, we have

$$\int_T^t \frac{\rho(s)(\omega(s)/\rho(s))^{\bullet}}{\Phi(1,\omega(s)/\rho(s))} ds \leq -\int_T^t \rho(s)q(s)ds - \int_T^t \left(\sqrt{\frac{k^*}{r(s)\rho(s)}}\omega(t) - \frac{1}{2}\sqrt{\frac{r(s)\rho(s)}{k^*}}p(s)\right)^2 ds + \frac{1}{4k^*}\int_T^t r(s)\rho(s)p^2(s)ds$$

Then, we get

$$\int_T^t \rho(s)\left[q(s) - \frac{1}{4k^*}r(s)p^2(s)\right] ds \leq -\int_T^t \frac{\rho(s)(\omega(s)/\rho(s))^{\bullet}}{\Phi(1,\omega(s)/\rho(s))} ds \qquad (2-4)$$

By the Bonnet's theorem, we see that for each $t \geq T$, there exists $a_t \in [T,t]$ such that

$$-\int_T^t \frac{\rho(s)(\omega(s)/\rho(s))^{\bullet}}{\Phi(1,\omega(s)/\rho(s))} ds = -\rho(T)\int_T^{a_t} \frac{(\omega(s)/\rho(s))^{\bullet}}{\Phi(1,\omega(s)/\rho(s))} ds \qquad \text{(2-5)}$$

From inequality (2-5) in inequality (2-4), the condition (3) and taking the limit superior on both sides, we obtain

$$\limsup_{t\to\infty}\int_T^t \rho(s)\left[q(s)-\frac{1}{4k^*}r(s)p^2(s)\right]ds \le -\rho(T)\limsup_{t\to\infty}\int_{\omega(T)/\rho(T)}^{\omega(a_t)/\rho(a_t)}\frac{du}{\Phi(1,u)}$$

$$\le -\rho(T)\limsup_{t\to\infty}\left[-\int_0^{\omega(T)/\rho(T)}\frac{du}{\Phi(1,u)}+\int_0^{\omega(a_t)/\rho(a_t)}\frac{du}{\Phi(1,u)}\right]$$

$$\le \rho(T)\limsup_{t\to\infty}\left(G\left(\frac{\omega(T)}{\rho(T)}\right)-G\left(\frac{\omega(a_t)}{\rho(a_t)}\right)\right)$$

$$\le \rho(T)\limsup_{t\to\infty}\left(G\left(\frac{\omega(T)}{\rho(T)}\right)+B^*\right)<\infty,$$

as $t\to\infty$, which contradicts to the condition (7). Hence the proof is completed.

Example2-2: Consider the following differential equation

$$\left(\frac{2\dot{x}(t)}{t^5+1}\right)^{\bullet}+\left(\frac{t^5+4t^5\cos t}{t^5+1}\right)\left(x^9(t)+\frac{x^{27}(t)}{x^{18}(t)+\left(2\dot{x}(t)/(t^5+1)\right)^2}\right)=\frac{x^9(t)\sin(x(t))}{t^2},\ t>0$$

Here $r(t)=\dfrac{2}{t^5+1},\ \psi\equiv 1,\ q(t)=\dfrac{t^5+4t^5\cos t}{t^5+1},\ g(x)=x^9,\Phi(u,v)=u+\dfrac{u^3}{u^2+v^2}$ and

$$\frac{H(t,x(t))}{g(x(t))}=\frac{\sin(x(t))}{t^2}\le\frac{1}{t^2}=p(t)\ \ \textit{for all}\ t>0\ \textit{and}\ x\neq 0.$$

Let $\rho(t)=\dfrac{t^5+1}{t^5}>0$ such that

$$\limsup_{t\to\infty}\int_T^t \rho(s)\left[q(s)-\frac{1}{4k^*}r(s)p^2(s)\right]ds=\limsup_{t\to\infty}\int_T^t \frac{s^5+1}{s^5}\left[\frac{s^5+4s^5\cos s}{s^5+1}-\frac{1}{4k^*}\left(\frac{2}{s^5+1}\right)\frac{1}{s^2}\right]ds=\infty.$$

We get all conditions of theorem2.2 are satisfied and hence every solution of the given equation is oscillatory. The numerical solutions of the given differential equation are found out using the Runge Kutta method of fourth order (RK4). We have

$$\ddot{x}(t)= f(t,x(t),\dot{x}(t))=x^9(t)\sin(x(t))-42.49\left(x^9(t)+\frac{x^{27}(t)}{x^{18}(t)+\dot{x}^2(t)}\right)$$

with initial conditions $x(1)=-0.5,\ \dot{x}(1)=1$ on the chosen interval $[1,100]$ and finding values of the functions *r, q* and f where we consider $H(t,x(t))=f(t)l(x)$ at *t=1, n=500* and *h=0.198.*

K	t_k	$x(t_k)$
1	1	-0.5
2	1.198	-0.302
3	1.396	-0.1039
4	1.594	0.0942
5	1.792	0.2922
6	1.99	0.4903
.	.	.
.	.	.
16	3.97	-0.1495
17	4.168	-0.3465
18	4.366	-0.5434
.	.	.
.	.	.
27	6.148	0.042
28	6.346	0.2334
29	6.544	0.4248

Table 2: Numerical solution of ODE2

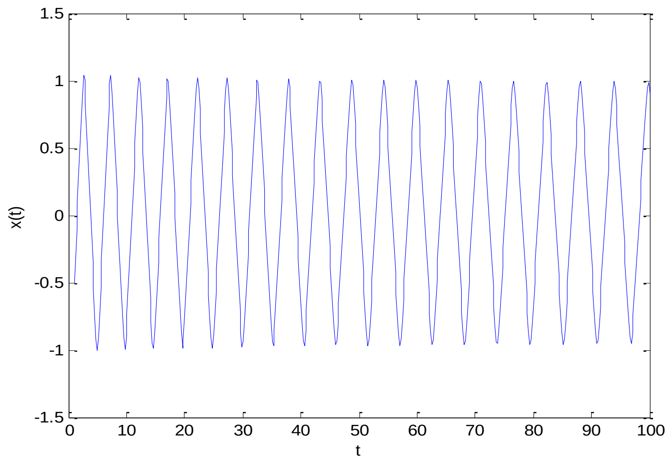

Figure2: Solution curve of ODE 2

Remark2.2:

If $(i)\ r(t) \equiv 1, (ii)\ \psi \equiv 1,\ (iii)\ \Phi(g(x(t)), r(t)\dot{x}(t)) \equiv \Phi(x(t), \dot{x}(t))\ and\ (iv)\ H(t, x(t)) \equiv 0,$ then theorem2.2 extends results of Bihari [2], Kartsatos [7]. All results of Bihari [2] and Kartsatos [7] can't be applied to the given equation in example2.2.

3. REFERENCES

[1] F. V. Atkinson, On second order nonlinear oscillations. Pacific. J. Math. 5 (1955), p. 643-647.

[2] I. Bihari, An oscillation theorem concerning the half linear differential equation of the second order, Magyar Tud. Akad.Mat. Kutato Int.Kozl. 8 (1963), p.275-280.

[3] W. B. Fite, Concerning the zeros of the solutions of certain differential equations, Trans. Amer. Math. Soc.19 (1918), p. 341-352.

[4] S. R. Grace and B. S. Lalli, Oscillation theorems for certain second perturbed differential equations, J. Math. Anal. Appl. 77 (1980), p. 205-214.

[5] J. R. Greaf, S. M. Rankin and P. W. Spikes, Oscillation theorems for perturbed nonlinear differential equations, J. Math. Anal. Appl. 65 (1978), p. 375-390.

[6] I.V. Kamenev, Integral criterion for oscillation of linear differential equations of second order, Math. Zametki 23 (1978), p. 249-251.

[7] A. G. Kartsatos, On oscillations of nonlinear equations of second order, J. Math. Anal. Appl. 24 (1968), p. 665-668.

[8] M. J. Saad, N. Kumaresan and Kuru Ratnavelu, Oscillation of Second Order Nonlinear Ordinary Differential Equation with Alternating Coefficients, Commu. in Comp. and Info. Sci. 283(2012), 367-373.

[9] M. J. Saad, N. Kumaresan and Kuru Ratnavelu, Oscillation Criterion for Second Order Nonlinear Equations With Alternating Coefficients, Amer. Published in Inst. of Phys. (2013).

[10] M. J. Saad, Ambarka A. Salhin and Fatima N. Ahmed, Oscillatory Behaviour Of second Order Nonlinear Differential Equations, International Journal Of Multidisciplinary Sciences and Advanced Technology. 1(2021), p. 565-571.

[11] P. Waltman, An oscillation criterion for a nonlinear second order equation, J. Math. Anal. Appl. 10 (1965), p. 439-441.

[12] A. Wintner, A criterion of oscillatory stability, Quart. Appl. Math. 7 (1949), p. 115-117.

[13] C. C. Yeh, Oscillation theorems for nonlinear second order differential equations with damped term, Proc. Amer. Math. Soc. 84 (1982), p. 397-402.

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