مجلة التربوي

مجلة علمية محكمة تصدر عن كلية التربية

جامعة المرقب

العدد الثالث عشر

يوليو 2018م

المجلة ترحب بما يرد عليها من أبحاث وعلى استعداد لنشرها بعد التحكيم .
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كافة الآراء والأفكار المنشورة تعبر عن آراء أصحابها ولا تتحمل المجلة تبعاتها .
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البحوث المقدمة للنشر لا ترد لأصحابها نشرت أو لم تنشر .

بحوث العدد

- Vocabulary knowledge and English reading obstacles faced by Libyan Undergraduate students at Elmergib University
- Difficulties Encountered by some Libyan Third – Year Secondary School Students in Forming and Using English Future Tenses
- An Acoustic Study of Voice On Investigating the difference between the effects of inductive and deductive approach in teaching grammar for sixth grade students in Anahda primary School
- Using Data Mining techniques in tracking the students' behavior in the asynchronous e-learning systems

Further Proof on Fuzzy Sequences on Metric Spaces

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Abstract

In this paper we prove that if a finite number of crisp sequences converges to some limit then there exists a fuzzy sequence converges to the same limit, and the convers is true**.**

Keywords: Metric space; Fuzzy sequence.

Introduction

The theory of fuzzy sets was introduced by Zadeh in 1965 [2], and since then there has been tremendous interest in the subject due to its diverse applications ranging from engineering and computer science to social behavior studies. Therefore, new concepts was introduced such as Fuzzy topological spaces in [3], Fuzzy metric spaces in [4].

As it is known, sequence in a metric space provides a natural framework for studying the paramount analyticities of a functions defined over them. Consequently, a new concept of fuzzy sequences in metric spaces, and convergent property established in [1].

The propose of this note is to prove the extension of theorem 8 stated in [1].

Preliminaries:

Definition: Let $X \neq \emptyset$, A sequence f in X is a function from N to X. i.e: $f: N \to X$.

Definition: Let $X \neq \emptyset$. A fuzzy sequence F in X is defined as $F: N \times X \to [0,1]$.

Example: Let $X = N$, $F: N \times N \to [0,1]$ such as that $F(n, x) = \frac{1}{2n+x^2}$ for all $n \in N$ and $x \in X$ .The following theorem gives the relation between the above concepts.

Theorem: Every crisp sequence in X is a fuzzy sequence in X.

Proof

let $X \neq \emptyset$. Let $S = \{ f: f \text{ is a crisp in } X\}$, and $A = \{F: F \text{ is a fuzzy sequence in } X\}$. Define $T: S \to F$. Now choose any $f \in S$ where $f: N \to X$ and consider $F(n, x) = \begin{cases} 1; & f(n) = x \\ 0; & \text{otherwise} \end{cases}$, which is a fuzzy sequence. Now let $f_1, f_2 \in S$ such that $f_1 \neq f_2$. Then there exists $n_\circ \in N$ where $f_1(n_\circ) \neq f_2(n_\circ)$. Let $f_1(n_\circ) = x$, and $f_2(n_\circ) = y$, which implies $F(n_\circ, x) = 1$ and $F(n_\circ, y) = 0$. Hence $T(f)_1 \neq T(f_2)$. Thus T is one-one. Therefore, every crisp sequence is a fuzz sequence.

Result: Converse of the previous theorem is not true.

Example : Let $X = Z$. Define $F: N \times X \to [0,1]$ as $F(n, x) = \frac{1}{n+x^2}$. It is obvious that F is a fuzzy sequence but it is not a crisp sequence. The following theorem gives some conditions that make a fuzzy sequence be a crisp sequence.

Theorem: Let $X \neq \emptyset$. A fuzzy sequence F on X is a crisp sequence if satisfies the following.

1. $F(n, x) = 0$ or 1 for all $n \in N$and for all $x \in X$.
2. For each $n \in N$, there exists unique x in X such that $F(n, x) = 1$.

Proof

Let F be a fuzzy sequence on X satisfies the given conditions. Using condition 2, we can define a crisp sequence f on as $f(n) = x$ if $F(n, x) = 1$. Now we attempt to show $F_f = F$. In case $F_f(n, x) = 1$ then $f(n) = x$. Then $F(n, x) = 1$. Hence $F_f(n, x) = F(n, x)$. If $F(n, x) = 0$ this means $f(n) \neq x$, implies $F(n, x) \neq 1$. therefor

$F(n,x) = 0$. Then $F_f(n,x) = F(n,x)$. Thus, there exists $f: N \to X$ as $F_f = F$. Hence F is a crisp sequence.

Convergence

Definition: Let (X,d) be a metric space and let F be a fuzzy sequence on X. Let $\alpha \in (0,1]$, $a \in X$. F is converges to a at level α if:

1. for each $n \in N$, there exists at least one $x \in X$ as $F(n,x) \geq \alpha$.
2. Given $\varepsilon > 0$, there is $n_\circ \in N$ such that $d(x,a) < \varepsilon$ for all $n \geq n_\circ$ and for all $x \in X$ with $F(n,x) \geq \alpha$.

Example: Take R with the usual metric.

Define $F: N \times X \to [0,1]$

as: $F = \begin{cases} 1; & x = \frac{1}{n} \\ 0; & \text{otherwise} \end{cases}$. The claim is $F \to 0$.

Take any $\alpha \in (0,1]$:

1. for each $n \in N$, $x = \frac{1}{x} \in IR$ as $F(n,\frac{1}{n}) \geq \alpha$.
2. Let $\varepsilon > 0$. Take $n_\circ \in N$ such that $n_\circ > {}^1\!/_\varepsilon$. Let $n \geq n_\circ$ and $F(n,x) \geq \alpha$. $d(x,a) = |x-a| = |x-0| = |x|$.

$n \geq n_\circ$ and $F(n,x) \geq \alpha \Rightarrow n > {}^1\!/_\varepsilon$ and $F(n,x) = 1 \Rightarrow n > {}^1\!/_\varepsilon$ and $x = \frac{1}{n}$.

Hence $d(x,a) = |x| = \left|\frac{1}{n}\right| = \frac{1}{n} < \varepsilon$. Given $\varepsilon > 0$ there exists $n_\circ \in N$ such that $d(x,0) < \varepsilon$ for all $n \geq n_\circ$ and for all $n \geq n_\circ$ and $x \in X$ with $n \geq n_\circ$ and $F(n,x) \geq \alpha$.

Theorem: Let f be a crisp sequence in a metric space X. Then f converges to 1 if and only if the fuzzy sequence F_f converges to 1 at some level $\alpha > 0$.

Proof

Let f be a crisp sequence in a metric space X. Then f can be considered as a fuzzy sequence F_f and let f converges to 1. Now take $\alpha > 0$. Let $\varepsilon > 0$. Therefore the crisp sequence f converges to 1, there exists $n_\circ \in N$ such that $d(x_n, 1) < \varepsilon$ for all $n > n_\circ$. Note that $F_f(n,x) = 0$ or $F_f(n,x) = 1$ for all $n \in N$, $x \in X$.

1. For each $n \in N, x = x_n$ as $F_f(n,x) = 1 \geq \alpha$.
2. When $n > n_\circ$ and $F_f(n,x) \geq \alpha$, gives $n > n_\circ$ and $F_f(n,x) \geq \alpha$. Which leads to $x = x_n$. Now $d(x,1) = d(x_n,1) < \varepsilon$. Given $\varepsilon > 0$, there exists $n_\circ \in N$ such that $n \geq n_\circ$ and $F_f(n,x) \geq \alpha$, implies $d(x,1) < \varepsilon$. Hence F_f converges to 1.

To prove the other side, let f be a crisp sequence in X. Let F_f be the corresponding fuzzy sequence . Let F_f converges to 1 at level $\alpha > 0$. Let $\varepsilon > 0$ be given.

Since F_f converges to 1 there exists $n_\circ \in N$ such $n \geq n_\circ$ and $F_f(n,x) \geq \alpha$, implies $d(x,1) \leq \varepsilon$.Now $F_f(n,x) \geq \alpha$ and $\alpha > 0$ implies that $F_f(n,x) = 1$, then $x = x_n$. Hence $d(x_n,1) < \varepsilon$.

Now, given $\varepsilon > 0$ there exists $n_\circ \in N$ such that $d(x_n,1) < \varepsilon$ for all $n \geq n_\circ$. Hence the crisp sequence f converges to 1.

Theorem: Let a_n, b_n be two crisp sequences in a metric space X. Let F be the fuzzy sequence defined as $F = \begin{cases} 1; & x = a_n \text{ or } x = b_n \\ 0; & \text{otherwise} \end{cases}$.

Then F converges to 1 at any level $\alpha > 0$ if and only if a_n, b_n converge to same limit 1.

Proof

Let a_n, b_n are crisp sequences converge to l. From the definition of F, for all $n \in N$ there exists $a_n \in X$ such that $F_f(n, a_n) = 1$, therefore $F_f(n, a_n) \geq \alpha$. Since a_n converges to 1 converges to , there exists $n_1 \in IN$ such that $d(a_n 1) < \varepsilon$ for all $n \geq n_1$. Since b_n converges to 1, there, exists $n_2 \in IN$ such that $d(b_n 1) < \varepsilon$ for all $n \geq n_2$. Let $n_\circ = \max\{n_1, n_2\}$. Now let $n \geq n_\circ$ and $F(n, a_n) \geq \alpha$ Since $\alpha > 0$, $F(n, x) \geq \alpha$, implies $F(n, x) \geq 1$, and hence $x = a_n$ or $x = b_n$. Since $n \geq n_1$, $d(a_n, 1) < \varepsilon$. Since $n \geq n_2$, $d(b_n, 1) < \varepsilon$. Hence $d(x, 1) < \varepsilon$. Therefor given $\varepsilon > 0$, there exists $n_\circ \in N$ such that $n \geq n_\circ$ and $F(n, x) \geq \alpha$ implies $d(x, 1) < \varepsilon$. Hence the fuzzy sequence F converges to 1 at any level $\alpha > 0$.

Conversely: Let $\varepsilon > 0$. As F converges to l, there is $n_\circ \in N$ such that $F(n, x) \geq \alpha$, $n \geq n_\circ$. Hence $d(x, l) < \varepsilon$. Let $n \geq n_\circ$, $F(n, x) = 1 \geq \alpha$. Therefore $d(a_n, l) < \varepsilon$. Thus a_n converges to l. The convergence of b_n can be proven in a Similar way.

An extension of the previous theorem is stated next.

Theorem: Let $\{ {a_n}^k : k \in \{1,2, \dots, m\}$ be a collection of crisp sequences in a metric space X . Let F be a fuzzy sequence in X defined as $F(n, x) = 1$ if $x = {a_n}^k$ for some k, and $F(n, x) = 0$ otherwise. converges if and only if for each $k, {a_n}^k$ converges to the same limit.

Proof

Let ${a_n}^k \to 1$; $k = \{1,2, \dots, m\}$. Let F be a fuzzy sequence in X defined as $F = \begin{cases} 1; & x = {a_n}^k \to l, k = \{1,2, \dots, m\} \\ 0; & \text{otherewise} \end{cases}$. By definition of F, for each $n \in N$ there exist ${a_n}^k$ for each k such that $F(n, {a_n}^k) = 1$. Hence $F(n, {a_n}^k) \geq \alpha$ for each k . Now let $\varepsilon > 0$ be given . Since ${a_n}^k$ converges to 1 for each k, there exists $n_k \in N$ such that $d({a_n}^k, 1) < \varepsilon$ for all $n \geq n_k$. Now let $n_\circ = \max\{n_k\}$. Now let $n \geq n_\circ$, $F(n, x) \geq \alpha$. Since $\alpha > 0$, $F(n, x) \geq \alpha$ implies $F(n, x) = 1$ and hence $x = {a_n}^k$ for some k . Since $n \geq n_k$ for each k, there $d({a_n}^k, 1) < \varepsilon$. Hence $d(x, 1) < \varepsilon$.

Converse: Let $\{{a_n}^k\}$ be a collection of crisp sequences in a metric space X, $k = \{1,2, \dots, m\}$. Let F be a fuzzy sequence defined as: $F = \begin{cases} 1; & x = {a_n}^k, \text{for some } k \\ 0; & \text{otherewise} \end{cases}$.

If F converges to 1 at some level $\alpha > 0$. Claim ${a_n}^k$ converges to 1 for all k. Let $\varepsilon > 0$ be given. Since F converges to 1, there exists $n_\circ \in IN$ such that $n \geq n_\circ$ and $F(n, x) \geq \alpha$ implies $d(x, 1) < \varepsilon$. Take $n_k \geq n_\circ$, $F(n, {a_n}^k) = 1 \geq \alpha$ For all k. Hence $d({a_n}^k, 1) < \varepsilon$ for all k. Let $n_\circ = \max\{n_k, k = 1,2 \dots, m\}$. Hence $d({a_n}^k, 1) < \varepsilon$. Hence, given $\varepsilon > 0$, there exists $n_\circ \in IN$ such that $d({a_n}^k, 1) < \varepsilon$ for all $n \geq n_\circ$. There for $\{{a_n}^k\}$ converge to 1 for all k.

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