# A Stochastic Optimisation Technique for Tuning a Continuous Stirred Tank Reactor Controllers

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**ABSTRACT**

A continuous stirred tank reactor mathematical model is developed based on the mass and energy balances for the reactor and heating system. A step change of the concentration is introduced and the temperature change in the reactor is measured. The objective of this paper is to comparatively study the application of PID, Generic Model Control, and Fuzzy logic controllers on the system and evaluate their performances according to the Integral of absolute error resulted. A simulated annealing algorithm is used to tune the controller's parameters. The control and simulation study has been implemented using MATLAB/SIMULINK.

**Keywords:** Mathematical modelling of continuous stirred tank reactor, MATLAB Simulation, PID controller, Generic Model Control, Fuzzy Logic Control, and Simulated Annealing .

## 1 Introduction

Continuous stirred tank reactor systems (CSTR) are the most important unit of a chemical plant used for unit operations. Basically a chemical reactor system has a complex nonlinear dynamic characteristic. There has been considerable interest in its state estimation and real time control based on mathematical modelling. However, the lack of understanding of the dynamics of the process, the highly sensitive and nonlinear behaviour of the reactor, has made it difficult to develop a suitable control strategy. An efficient control of the CSTR can be achieved only through an accurate model [1].

A PID controller represents the simplest form of controller that utilises Derivative and Integral operations on the system. PID controllers have several important functions: they have the ability to eliminate steady-state error through the integral action, and they can cope with actuator saturation, if used with anti-windup. These controllers are also effective for many control problems, particularly where there are a benign process dynamics and modest performance requirements [2]. PID controller can be represented by the following equation.

$$u(t) = K_c\left(\varepsilon(t) + \frac{1}{\tau_i}\int_0^t e(t)\,dt + \tau_D\frac{d\varepsilon(t)}{dt}\right) \qquad (1)$$

Where: $K_c$ is Proportional constant, $\tau_i$ is Time integral constant, $\tau_D$ is Derivative time constant, $\varepsilon$ error, and u is the controller output The need for improved process control has become obvious in recent years. Since 1987, there have been growing interest in the use of generic model control (GMC), which has been exposed to have certain robustness for a wide range of process nonlinearity against model mismatches [3]. The desired response can be obtained by incorporating two tuning parameters. More details of GMC method can be found in [4]. Consider a process described by the following equation:

$$x = f(x, u, d, t) \tag{2}$$

$$y = g(x) \tag{3}$$

Where x is the state variable, u is the manipulated variable, d is the disturbance variable t is the time, and y is the output. In general, f and g are some nonlinear functions. It follows from (2) and (3) that:

$$y = G_x f(x, u, d, t) \tag{4}$$

For a specific desired steady state value, the GMC algorithm specifies a rate of change of the output variables as:

$$y = K_1(y_{sp} - y) - K_2 \int (y_{sp} - y) dt \tag{5}$$

In (5), two process desires are obvious. First, when the system is at a greater distance from the setpoint, then the system should travel towards the setpoint more quickly. Moreover, the longer that the system has remained offset from the setpoint, then the system should also travel towards the setpoint more quickly. The values of K1 and K2 are what determine the speeds. Therefore, to solve for the control, the actual output rate is set equal to the desired output rate, in other words setting (4) equal to (5), result in the following equation from which the control, u, can be solved.

$$G_x f(x, u, d, t) = K_1(y_{sp} - y) - K_2 \int (y_{sp} - y) dt \tag{6}$$

Fuzzy Logic Control has emerged as one of the most active and fruitful areas [5,6]. FLC is based on a spirit that is close to human thinking, and natural language, where the essential part of fuzzy logic is a set of linguistic control rules related by the dual concepts of fuzzy implication and compositional rules of inference [7]. FLC differs from conventional control methods, it incorporates a simple rule-based approach to solve the control problem rather than modelling the system mathematically. It also uses imprecise data, but descriptive of what must happen [8]. Figure 1, shows typical MFs of the controller. Hence the number of MFs used for variable is 3, then the number of rules required to map the input into the output is 3.

## 2 mathematic model of the continuous stirred tank reactor

A mathematical model of a continuous stirred tank reactor is developed depending on mass and energy balances. A summing first order irreversible exothermic reaction (A → B) in a Continuous Stirred Tank Reactor as shown in Figure 2. The heat generated by the reaction is removed using a cooling coil inside the reactor. Perfectly mixing is assumed in CSTR and the change in volume due to reaction is negligible. The reactor mass and energy equations are:

Over all mass balance

$$\frac{\mathrm{dV}}{\mathrm{dt}} = \mathrm{F_i} - \mathrm{F} \quad \text{and} \quad F_i = F \tag{7}$$

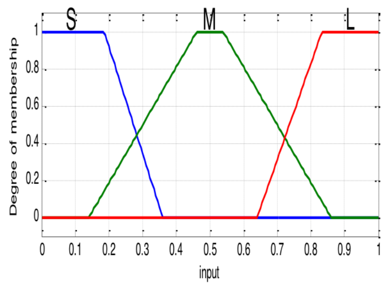


**Figure 1** *Typical Membership Functions*

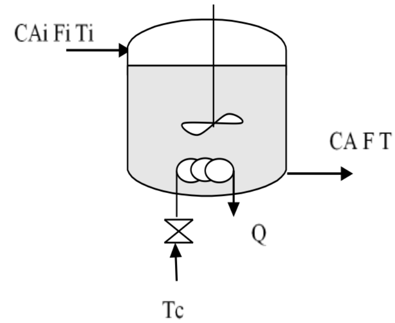


**Figure 2** *Continuous Stirred Tank*

$F_i, F$ are inlet, outlet flow, V reactor volume, t is the time, $C_{Ai}, C_A$ inlet, outlet concentration of component A, $T_i, T$ inlet, outlet temperature, r is reaction rate, E is activation energy, R is gas constant, $k_0$ is pre-exponential constant, $\rho$ is density, $C_p$, specific heat capacity, $H_r$ heat of reaction, $T_c$ coolant temperature, and UA is a product of heat transfer coefficient and area.

Component (A) mass balance

$$\frac{dVC_A}{dt} = F_i C_{Ai} - FC_A - rV \tag{8}$$

Where r is the rate of a first order reaction

$$r = k_0 e^{\frac{-E}{RT}} C_A \tag{9}$$

and V is constant then (8) can written as:

$$\frac{dC_A}{dt} = \frac{F}{V} C_{Ai} - \frac{F}{V} C_A - k_0 e^{\frac{-E}{RT}} C_A \tag{10}$$

Heat balance

$$\frac{\rho dVC_p T}{dt} = \rho C_p F_i T_i - \rho C_p FT - H_r VC_A k_0 e^{\frac{-E}{RT}} - UA(T - T_c) \tag{11}$$

Where, V is constant, and the specific heat $C_p$ is a function of temperature then from (8), and (11).

$$\frac{dT}{dt} = \frac{F}{V}(T_i - T) - \frac{H_r C_A k_0 e^{\frac{-E}{RT}}}{\rho C_p} - \frac{UA}{\rho C_p V}(T - T_c) \tag{12}$$

## 3 simulated annealing and its application to controller tuning

Simulated annealing is a global search method that is based on the analogy with the physical annealing process of solids [9, 10, 11]. This optimisation technique has been applied to a CSTR for tuning proportional integral (PI), generic model (GMC), and Fuzzy controllers that are used to control the temperature and the concentration of the process, in MATLAB and SIMULINK environment. More detail on Simulated annealing can be found in [12].

## Simulation Results

The feedback control system can be represented in a Simulink as shown in Figure 3 The performance of the three types of controllers are illustrated below. Figures 4, 5, 6 shows the results obtained by conventional settings. However, when applying the stochastic simulated annealing optimization method, the best values of the IAE obtained are 0.1791, 0.1693, 0.2048 for PI, GMC and fuzzy Controllers respectively. Where, the number of investigated

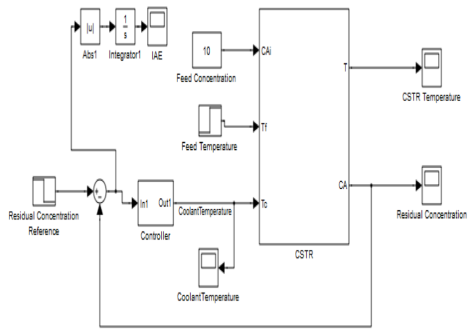


**Figure** 3 *Feedback control system*

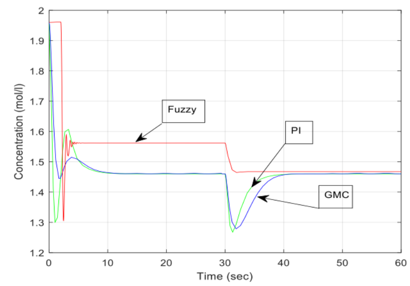


**Figure** 4 *Concentration response of different controllers by conventional*

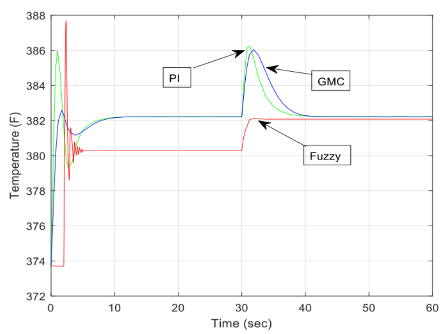


**Figure** 5 *Temperature response of different Controllers by conventional*

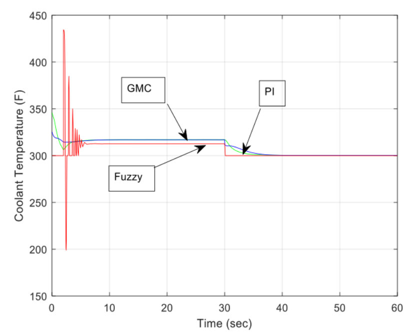


**Figure** 6 *Coolant Temperature response for different Controllers by*

solutions used for PI and GMC are 1000, while for Fuzzy controller are 3500 as there are 8 points to be tuned for both input and output membership functions. However, the best solutions where found at a simulation times 905, 655, 548 for PI, GMC, and Fuzzy controllers respectively. Figures 7,8,9,10,11, 12 and 13 depict the results obtained using (SA) algorithm.

It is obvious that for both PI and GMC controllers an acceptable result can be achieved using conventional tuning methods, but it is very difficult to have a good membership function setting for fuzzy controller using trial and error. While, when applying simulated annealing the performance of the controllers in tracking the step change of the concentration from its initial value of 1.96 to 1.46 mol/l has been achieved. However, the controllers have the capability of eliminating the effect of the feed temperature disturbance from 300 F to 305 F on the concentration which is obvious at 30 sec as can be seen in figures 8, and 9. Moreover, it can be clearly seen in figures 10, and 12 that the Temperature and the coolant Temperature (Controller output) responses are changing according to their dependency to the concentration change, where, it is realized that at the initial concentration value, the temperature is 373.72 F, and the coolant Temperature is 300 F. When the concentration step change introduced at time 2 sec where it has been reduced to 1.46 mol /l, the temperature

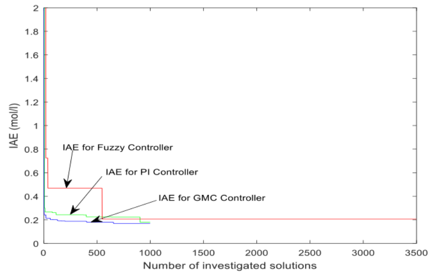


**Figure**. **7** *IAE obtained by SA for using different controllers*

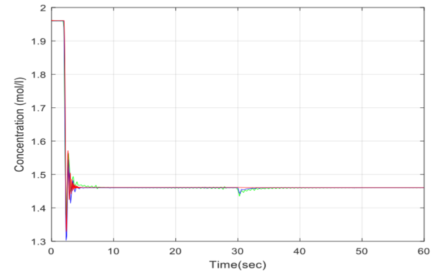


**Figure** 8 *Concentration response of different controllers*

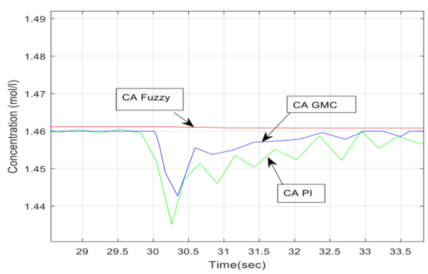


**Figure 9** *Enlargement of Concentration response of different controllers*

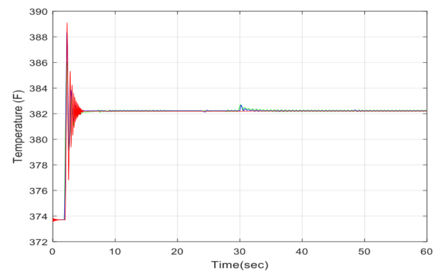


**Figure 10** *Temperature response of different Controllers*

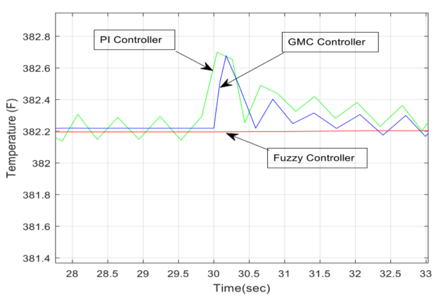


**Figure 11** *Enlargement of Temperature response of different Controllers*

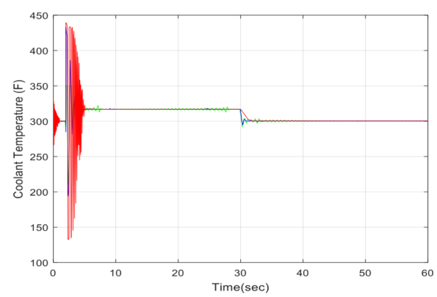


**Figure 12** *Coolant Temperature response for different Controllers*

value rose to 382.22 F as well as the coolant temperature that rose to 316.9 F. However, at 30 sec when the feed temperature disturbance was added, the controllers quickly overcame the disturbance and brought the temperature back to its steady state value, while the coolant temperature has dropped to 300.3 F which is the required controller value to keep the controlled parameter at its desired value. It is obvious that fuzzy controller response is a bit oscillatory at the start of the step change. Moreover, the fuzzy controller has better overcome of the feed temperature disturbance than the PI and the GMC controllers although it is a bit slower, but on other hand they are much better in eliminating the steady state error. The following table shows the results obtained when tuning the controllers using conventional methods available in MATLAB optimization toolboxes and simulated annealing optimization technique illustrated above.

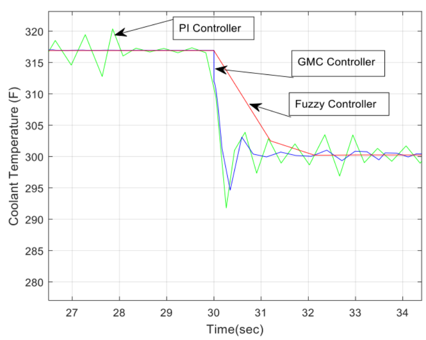


**Figure 13** *Enlargement of Coolant Temperature response for different*

**Table 1.** *Simulation results*

| Using Conventional methods | |
|---|---|
| Controller Type | IAE (mol/l) |
| PI | 1.8992 |
| GMC | 1.8606 |
| Fuzzy | 4.2093 |
| Using SA algorithm | |
| PI | 0.1791 |
| GMC | 0.1693 |
| Fuzzy Logic | 0.2048 |

## 4 Conclusion

Fuzzy Logic Controllers are nonlinear and have the influence of rejecting the disturbances better than the PI, and GMC controllers. Moreover, PI and GMC controllers have the inherent character of eliminating the steady state error which is unbeatable. The table above shows that simulated annealing is a powerful stochastic optimisation search method, where by comparing The IAE obtained using this algorithm to that obtained from conventional methods, it can be clearly seen that simulated annealing has found the best possible parameters that minimise the IAE to its minimum values which gives a better result of controller performance.

## References

[1] Suja Malar & Thyagarajan, "Modelling of Continuous Stirred Tank Reactor Using Artificial Intelligence Techniques", Department of Electrical and Electronics Engineering, PET Engineering College of India, Vol.3, pp. 145-155, (2009).

[2] WG da Silva; PP Acarnley; JW Finch; ‘Tuning of brushless dc drive speed controller with an on-line genetic algorithm’, European Power Electronics Conference, Lausanne, 1999.

[3] P. L. Lee, G. R. Sullivian. Generic model control (GMC). Computers and Chemical Engineering, 1988, 12(6): 573-580.

[4] B. J. Cott, S. Macchietto. Temperature control of exothermic batch reactors using generic model control. Industrial Engineering Chemical Research, 1989, 28(8): 1177-1184.

[5] CP. Coleman, D. Godbole: ‘A comparison of robustness: fuzzy logic, PID, and sliding mode control’, Proceedings of the Third IEEE Conference on Fuzzy Systems, V. 3, P. 1654 -1659, 1994

[6] WG da Silva, PP Acarnley, and JW Finch; ‘A comparison of fuzzy and conventional speed control in electric drives’, European Power Electronics Conference, Lausanne, 1999.

[7] N. Gulley, and J. S. Roger Jang, ‘Fuzzy logic toolbox for use with MATLAB’, The Math Workers Inc., 1995-1999.

[8] A. Haj-Ali, H. Ying; ‘Structural analysis of fuzzy controllers with nonlinear input

[9] Prof. Song Prof. Irving;” Optimisation techniques for electrical power systems’’ part 2 Heuristic optimisation methods; Power Engineering Journal, P.151-160, June 2001

[10] X. Yao;’A new simulated annealing algorithm’, Int. J. Computer Math. V. 56, P.161-168, 1995.

Z. Xiaoyum, H. Yun, D. John; ‘Genetic algorithms and simulated annealing for robustness analysis’, Proceedings of the American Control Conference, V. 6, P. 3756-3760, 1997.

[11] Z. Xiaoyum, H. Yun, D. John; ‘Genetic algorithms and simulated annealing for robustness analysis’, Proceedings of the American Control Conference, V. 6, P. 3756-3760, 1997.

[12] Y. Alsadiq ‘A Comprehensive Tuning of Distillation Column Composition Controllers using Simulated Annealing ‘, Int'l Conference on Artificial Intelligence, Energy and Manufacturing Engineering (ICAEME’2014), P. 35- 41, June 9-10, 2014 Kuala Lumpur (Malaysia)