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جامعة المرقب

العدد العشرون
يناير 2022م

هيئـــة تحريـر
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- المجلة ترحب بما يرد عليها من أبحاث وعلى استعداد لنشرها بعد التحكيم .
- المجلة تحترم كل الاحترام آراء المحكمين وتعمل بمقتضاها .
- كافة الآراء والأفكار المنشورة تعبر عن آراء أصحابها ولا تتحمل المجلة تبعاتها .
- يتحمل الباحث مسؤولية الأمانة العلمية وهو المسؤول عما ينشر له .
- البحوث المقدمة للنشر لا ترد لأصحابها نشرت أو لم تنشر .

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يشترط في البحوث العلمية المقدمة للنشر أن يراعى فيها ما يأتي :

- أصول البحث العلمي وقواعده .
- ألا تكون المادة العلمية قد سبق نشرها أو كانت جزءا من رسالة علمية .
- يرفق بالبحث تزكية لغوية وفق أنموذج معد .
- تعدل البحوث المقبولة وتصحح وفق ما يراه المحكمون .
- التزام الباحث بالضوابط التي وضعتها المجلة من عدد الصفحات ، ونوع الخط ورقمه ، والفترات الزمنية الممنوحة للتعديل ، وما يستجد من ضوابط تضعها المجلة مستقبلا .

تنبيهات :

- للمجلة الحق في تعديل البحث أو طلب تعديله أو رفضه .
- يخضع البحث في النشر لأولويات المجلة وسياستها .
- البحوث المنشورة تعبر عن وجهة نظر أصحابها ، ولا تعبر عن وجهة نظر المجلة .

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An Active-Set Line-Search Algorithm for Solving Multi-Objective Transportation Problem

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Abstract

In this paper, an algorithm is proposed to solve multi-objective transportation (MOT) problem, where the MOT problem converting to a single-objective constrained optimization (SOCO) problem by using weighted method. In the proposed algorithm an active set strategy is used together with multiplier method to transform SOCO problem to unconstrained optimization problem and we used an active-set line-search algorithm to solve it. In this work, the effect of changing weights on MOT problem is studied to show the degree of satisfaction of each objective. We also make a comparative study between our proposed approach and different approaches treated the multi-objective transportation problem before. The proposed approach is carried out on two multi-objective transportation test problems.

***Key Words**: Multi-objective transportation problem, Line-search, weighting approach.*

1 Introduction

Transportation problem is an optimization problem involves the transportation or physical distribution of goods from several supply points to various destinations in such a way so that the total transportation cost is minimum. When a transportation problem involves more than one objective function, the task of finding one or more optimal solutions is known as multi objective transportation problem. For multiple conflicting objectives, there cannot be a single optimum solution which simultaneously optimizes all the objectives. The resulting outcome is a set of optimal solutions with varying degree of objective values. The multi-objective transportation problem is of great interest to many researchers and several local methods have been proposed to solve it (see, [16], [17], [22], [23]).

In this work, we convert the MOT problem to a SOCO problem by using a weighting approach. The oldest and most widely used method of combining objective, proposed by Zadeh [27]. Ease of use and user preferences are advantages to reflect on the model. All effective solution to generate value in the field and has the condition of being convex objective functions

In this paper an active set strategy is used together with the multiplier method. The general idea behind the active-set strategy is to identify at every iteration, the active inequality constraints and treat them as equalities. This allows the use of the well-developed techniques for solving the equality constrained optimization problems. Many authors have proposed active-set algorithms for solving a general nonlinear programming problem (see, [12-15], [19]). The main idea of the multiplier methods is to replace the equality constrained optimization problem with a sequence of unconstrained optimization problem and at the same time the penalty parameter needs not to go to infinity (see, [9], [21]).

A line-search globalization strategy is used to modify the local method in such a way that it is guaranteed to converge at all even if the starting point is far from the solution and improves the solution quality for the same approach. Line search method is iterative method. The iterations choose a search direction by moving along the direction while taking an appropriate step size. Line-search rules can be classified into two types, exact line search rules and inexact line search rules. Many researchers believe the exact line search is time-consuming to be carried out or impossible to find in practical computation. Therefore we opt to use the inexact line searches to identify the step size that will ensure a substantial reduction in function at minimum cost. Many inexact line-search methods have been proposed (see, [1], [18], [25]) and others.

Here, we introduce some notations for subscripted functions denote function values at particular points; for example, $f_k = f(x_k)$, $\nabla f_k = \nabla f(x_k)$, $L_k = L(x_k, \lambda_k)$, $\nabla_x L_k = \nabla_x L(x_k, \lambda_k)$, and so on. The matrix H_k denotes the Hessian of the objective function at the point (x_k) or an approximation to it. Finally, all norms are l_2-norms.

The paper is organized as follows. In section 2, presents some preliminaries of MOT problem and some definitions. Section 3, the proposed algorithm is presented. Section 4 contains of a numerical illustration of the proposed approach , two examples are solved and compare our results with the other methods. Finally, Section 5 contains concluding remarks.

2. Preliminaries

2.1 Mathematical Formulation of MOT Problem.

In MOT problem the product is to be transported from m sources to n destination points. The cost of transporting a unit form source i to destination j is also denoted as c_{ij}, this can be considered to be delivery time, cost of damage, or safety of delivery, etc. A variable x_{ij} represents the unknown quantity to be shipped from source i to destination j. Let their capacities be $a_1, a_2, \dots, a_m$ and $b_1, b_2, \dots, b_n$, respectively. The objectives are to minimize the total cost of transportation, delivery time, and/or damage cost. Let $f_1, f_2, \dots, f_p$ be p objectives which are to be minimized. With these assumptions, the MOT problem can be formulated as follows:

$$\begin{aligned} &minimize \quad f_k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}\,, k = 1,2,\dots,p \\ &subject\ to \quad \sum_{j=1}^{n} x_{ij} = a_i, i = 1,2,\dots,m, \\ &\sum_{i=1}^{m} x_{ij} = b_j, \ \ j = 1,2,\dots,n, \\ &x_{ij} \geq 0, \ \ i = 1,2,\dots,m, j = 1,2,\dots,n, \end{aligned} \tag{1}$$

2.2 Definitions of solution concept

In this subsection, some definitions regarding the solution concept of the MOT problem (1) are introduced.

Definition 1 (*Pareto optimal Solution* [20])

A feasible decision vector $x^* = \{x_{ij}^*\} \in S$ *is Pareto optimal solution, a non-dominated or efficient solution for MOT problem (1) iff there does not exist another feasible decision vector* $x = \{x_{ij}\} \in S$ *such that*

- $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} \leq \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}^{*} \quad for\ all\ k = 1,2,\dots,p$.
- $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} < \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}^{*} \quad for\ at\ least\ one\ index\ k.$

Definition 2 (*Compromise Solution* [20])

A compromise solution of the MOT problem (1) is a feasible solution which is preferred by the DM over all other feasible solutions, taking into consideration all criteria contained in the multi-objective functions.

3. The Proposed Approach

In this section, the proposed algorithm is presented. The proposed algorithm contains two stages initialization stage and active-set line-search algorithm stage.

3.1. Initialization stage

- *Converting MOT problem (1) to SOCO problem:*

  By using the weighting approach, the MOT problem (1) is converted to the following SOCO problem with equality and inequality constraints problem

$$\begin{aligned} &\textit{minimize} \quad F = \sum_{k=1}^{p}\sum_{i=1}^{m}\sum_{j=1}^{n} w_k c_{ij}^{k} x_{ij}\,, \\ &\textit{subject to} \quad \sum_{j=1}^{n} x_{ij} = a_i, i = 1,2,\dots,m, \\ &\qquad\qquad \sum_{i=1}^{m} x_{ij} = b_j,\ \ j = 1,2,\dots,n, \\ &\qquad\qquad x_{ij} \geq 0,\quad i = 1,2,\dots,m, j = 1,2,\dots,n, \end{aligned} \tag{2}$$

where $\sum_{k=1}^{p} w_k = 1$ and $w_k \geq 0$ for all k.

The above problem can be written as follows:

$$\begin{aligned} &\textit{minimize} \quad F(x) \\ &\textit{subject to} \quad y(x) = 0, \\ &\qquad\qquad z(x) \leq 0, \end{aligned} \tag{3}$$

Where $y(x) = \left[\sum_{j=1}^{n} x_{ij} - a_i, \sum_{i=1}^{m} x_{ij} - b_j\right]$ and $z(x) = [x_{ij}]^T$, $i = 1,2,\dots,m$, $j = 1,2,\dots,n$. The function $F: R^{n\times m} \to R$, $y: R^{n\times m} \to R^{n+m}$, and $z: R^{n\times m} \to R^{n\times m}$ are twice continuously differentiable.

- *Converting the SOCO problem with equality and inequality constraints problem (3) to equality constrained problem:*

Following the active set strategy in [10], we define a 0-1 diagonal indicator matrix $U(x) \in R^{(n\times m)\times(n\times m)}$, whose diagonal entries are

$$u_e(x) = \begin{cases} 1 \ if \ z(x) \geq 0, \\ 0 \ if \ z(x) < 0, \end{cases}$$

Using the above matrix, we transform Problem (3) to the following equality constrained optimization problem

$$\begin{aligned} &minimize \quad F(x) \\ &subject\ to \quad y(x) = 0, \\ &\frac{1}{2}\tilde{z}(x)U(x)\tilde{z}(x) = 0, \end{aligned} \qquad (4)$$

The above problem can be rewritten as:

$$\begin{aligned} &minimize \quad F(x) \\ &subject\ to \quad Y(x) = 0, \end{aligned} \qquad (5)$$

where $Y(x) \in R^{n+m}$ such that $Y(x) = \left[y(x) \ \ \frac{1}{2}\tilde{z}(x)U(x)\tilde{z}(x)\right]$.

The Lagrangian function associated with the above problem is the function

$$L(x,\lambda) = F(x) + \lambda^T Y(x), \qquad (6)$$

where $\lambda \in R^{n+m}$ is the Lagrangian multiplier vector associated with the equality constraint $Y(x)$.

Using an augmented Lagrangian method, we transform the equality constrained optimization problem (5) to the following unconstrained optimization problem

$$minimize\ \emptyset(x,\lambda;r) = L(x,\lambda) + \frac{r}{2}\|Y(x)\|^2$$

$$subject\ to \qquad x \in R^{n\times m} \qquad (7)$$

where r is a parameter usually called the **penalty parameter.**

3.2 Active-set line-search algorithm stage

In this subsection, we present the description of the active set line-search algorithm which is used to solve the single-objective problem (7). Then we present the main algorithm that is used to solve the MOT problem.

3.2.1 Computing a search direction d_k

At the point x_k, we used a quasi-Newton method to find the search direction d_k which is minimizes the quadratic model

$$m_k(d) = L_k + \nabla_x L_k^T d + \frac{1}{2} d^T B_k d + \frac{r_k}{2}\left\|H_k + \nabla Y_k^T d\right\|^2, \qquad (8)$$

where B_k is the Hessian matrix of the Lagrangian function (6) or an approximation to it. If $(B_k + r_k \nabla Y_k \nabla Y_k^T)$ is a positive definite matrix, then $x_k + d_k$ uniquely minimizes the quadratic form (8) where d_k satisfies

$$(B_k + r_k \nabla Y_k \nabla Y_k^T)d = -(\nabla_x L_x + r_k \nabla Y_k Y_k) \qquad (9)$$

Since we always require, for all k, the quasi-Newton direction d_k be a descent direction, i.e

$$\nabla_x \emptyset(x_k, \lambda_k; r_k)^T d_k \le 0, \qquad (10)$$

then the matrix $(B_k + r_k \nabla Y_k \nabla Y_k^T)$ must be positive definite (see, [11]). To check the matrix $(B_k + r_k \nabla Y_k \nabla Y_k^T)$ is positive definite, we useTarazaga's condition (see, [26]). This condition says that, if

$$trace(B_k + r_k \nabla Y_k \nabla Y_k^T) - (n-1)^{\frac{1}{2}} \left\| B_k + r_k \nabla Y_k \nabla Y_k^T \right\|_F > 0,$$

then $B_k + r_k \nabla Y_k \nabla Y_k^T$ is positive definite. Otherwise, we update the diagonal of the matrix $(B_k + r_k \nabla Y_k \nabla Y_k^T)$ by adding a positive large number ρ to the diagonal and compute the search direction d_k by solving

$$(B_k + r_k \nabla Y_k \nabla Y_k^T + \rho_k I)d = -(\nabla_x L_x + r_k \nabla Y_k Y_k) \qquad (11)$$

3.2.2 Computing a step length α_k

Once the descent direction d_k is determined we compute a step length α_k along the descent direction and set $x_{k+1} = x_k + \alpha_k d_k$. We used the backtracking line-search to find the step length α_k, where α_k satisfies the sufficient decrease condition

$$\emptyset(x_k + \alpha_k d_k, \lambda_k; r_k) \le \emptyset(x_k, \lambda_k; r_k) + \sigma \alpha_k \nabla\emptyset(x_k , \lambda_k; r_k)^T d_k, \qquad (12)$$

where $\sigma \in (0,1)$ is a fixed constant (see [1]). This process is summarized in the following algorithm.

Algorithm 1 *(Backtracking Line-Search Algorithm)*

Step 0. *(Initialization)*

Given $\sigma \in (0,1)$, $\eta \in (0,1)$, *and set* $\alpha_k = 1$.

Step 1. *While* $\emptyset(x_k + \alpha_k d_k, \lambda_k; r_k) > \emptyset(x_k, \lambda_k; r_k) + \sigma \alpha_k \nabla\emptyset(x_k , \lambda_k; r_k)^T d_k$

set $\alpha_k = \eta \alpha_k$.

End while.

Step 2. *Set* $x_{k+1} = x_k + \alpha_k d_k$.

3.2.3 Updating λ_{k+1} and r_k

Once x_{k+1} computed, we update the Lagrange multiplier λ_k. To estimate the Lagrangian multiplier vector λ_{k+1} we solve

$$minimize_{\lambda \in R^{n+m}} \|\nabla f_{k+1} + \nabla Y_{k+1} \lambda\|^2$$

After updating the Lagrangian multiplier, the penalty parameter is updated. To update r_k , we use the scheme that was proposed by Bertsekas (1995) [8]. The adjustment scheme is to increase r_k by multiplication with a factor $\zeta > 1$ only if the constraint violation as measured by $\|Y_{k+1}\|$ is not decreased by a factor $\gamma < 1$ over the previous minimization; i.e.,

$$r_{k+1} = \begin{cases} \zeta r_k & if \ \ \|Y(x_{k+1})\| > \gamma \ \|Y(x_k)\|, \\ r_k & if \ \ \|Y(x_{k+1})\| \le \gamma \ \|Y(x_k)\|, \end{cases} \tag{13}$$

Finally, the algorithm is terminated when $\|\nabla_x L_k\| + \|\nabla Y_k Y_k\| \le \varepsilon_1$, or $\|d_k\| \le \varepsilon_2$, for some $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$.

3.2.4 Active set line-search algorithm

The main steps of our active-set line-search algorithm are explained in detail as follows:

Algorithm 2 *(Active-Set Line-Search Algorithm)*

Step 0. *(Initialization)*

Given $x_0 \in R^{n\times m}$. *Compute* U_0 *and* λ_0 . *Choose* $0 < \sigma < 1, 0 < \eta < 1$,

$\gamma > 0, \zeta > 0, \varepsilon_1 > 0,$ *and* $\varepsilon_2 > 0$. *Set* $r_0 = 1$ *and* $k = 0$.

Step 1. *If* $\|\nabla_x L_k\| + \|\nabla Y_k Y_k\| \le \varepsilon_1$. *Then terminate the algorithm.*

Step 2. *Compute the search direction* d_k *by solving (9).*

Step 3. *If* $\nabla_x \emptyset(x_k, \lambda_k; r_k)^T d_k < \varepsilon_1$, *then go to Step 5.*

Else, set $\rho_k = 10^3$.

While $trace(B_k + r_k \nabla Y_k \nabla Y_k^T) - (n-1)^{\frac{1}{2}} \left\| B_k + r_k \nabla Y_k \nabla Y_k^T \right\|_F \le \varepsilon_1$.

Set $\rho_k = 2\rho_k$.

End while.

End if.

Step 4. *Compute the search direction* d_k *by solving (11).*

Step 5. *If* $\|d_k\| \leq \varepsilon_2$*. Then terminate the algorithm.*

Step 6. *Compute the step length* α_k *as follows*

a) Set $\alpha_k = 1$.

While $\emptyset(x_k + \alpha_k d_k, \lambda_k; r_k) > \emptyset(x_k, \lambda_k; r_k) + \sigma\alpha_k \nabla\emptyset(x_k, \lambda_k; r_k)^T d_k$

Set $\alpha_k = \eta\alpha_k$.

End while.

b) Set $x_{k+1} = x_k + \alpha_k d_k$.

Step 7. *Compute* U_{k+1}.

Step 8. *Compute the Lagrangian multiplier* λ_{k+1} *by solving*

$$minimize_{\lambda \in R^{n+m}} \|\nabla f_{k+1} + \nabla Y_{k+1}\lambda\|^2$$

Step 9. *To update the penalty parameter* r_k.

If $\|Y(x_{k+1})\| > \gamma \|Y(x_k)\|$

then set $r_{k+1} = \zeta r_k$.

Else, set $r_{k+1} = r_k$.

End if.

Step 10. *Set* $k = k + 1$ *and go to Step 1.*

4 Numerical Illustrations

In this section, we introduce two illustrative examples two examples of MOT problem are considered. The proposed algorithm was implemented on 2.7 MHZ PC using MATLAB 7.9 to confirm the effectiveness of the algorithm.

4.1 Implementation details

For implementing the proposed approach, the parameters have been selected as follows: $\sigma = 0.1$, $\eta = 0.5$, $\zeta = 2$, $\gamma = 0.25$. Successful termination with respect to our line-search algorithm means that the termination condition of the algorithm is met with $\varepsilon_1 = 10^{-6}$ and $\varepsilon_2 = 10^{-8}$. On the other hand, unsuccessful termination means that the number of iterations is greater than 500, the number of function evaluations is greater than 800.

4.2 Example 1

Consider the MOT problem mentioned in ([2-4], [7], [15], [28]). To illustrate the application of the proposed algorithm. The problem has the following characteristics:

$$c^1 = \begin{bmatrix} 1 & 2 & 7 & 7 \\ 1 & 9 & 3 & 4 \\ 8 & 9 & 4 & 6 \end{bmatrix} \quad \text{and} \quad c^2 = \begin{bmatrix} 4 & 4 & 3 & 4 \\ 5 & 8 & 9 & 10 \\ 6 & 2 & 5 & 1 \end{bmatrix}$$

with Supplies $a_1 = 8$, $a_2 = 19$, and $a_3 = 17$ and demands $b_1 = 11$, $b_2 = 3$, $b_3 = 14$, and $b_4 = 16$.

The mathematical programming model of the above problem is written as follows:

$$minimize\ \ f_1 = x_{11} + 2x_{12} + 7x_{13} + 7x_{14}$$
$$+x_{21} + 9x_{22} + 3x_{23} + 4x_{24}$$
$$+8x_{31} + 9x_{32} + 4x_{33} + 6x_{34},$$
$$minimize\ \ f_2 = 4x_{11} + 4x_{12} + 3x_{13} + 4x_{14}$$
$$+5x_{21} + 8x_{22} + 9x_{23} + 10x_{24}$$
$$+6x_{31} + 2x_{32} + 5x_{33} + x_{34},$$
$$subject\ to \quad x_{11} + x_{12} + x_{13} + x_{14} = 8,$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 19,$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 17,$$
$$x_{11} + x_{21} + x_{31} = 11,$$
$$x_{12} + x_{22} + x_{32} = 3,$$
$$x_{13} + x_{23} + x_{33} = 14,$$

$$x_{14} + x_{24} + x_{34} = 16,$$

$$x_{ij} \geq 0, \quad i = 1,2,3, \qquad j = 1,2,3,4.$$

4.2.1 Results and Discussions of Example 1:

A weighting approach is used together with the active-set line search algorithm (2) to solve the above problem. The solution obtained for different weights is given in Table 1 and by discussing the effect of changing weights on the two objective functions f_1 and f_2, we note from Figure (1) that the best value of w_1 is 0.4 and w_2 is 0.6, which give $f_1 = 177$ and $f_2 = 178$ as the best compromise solution.

To evaluate the performance of the suggested approach we compare the best results with the recently reported methods are given in Table 2.

Table 1. Compromise objective values corresponding to priorities

	Weights assigned	f_1 , f_2
1	$w_1 = 0.0$, $w_2 = 1.0$	188 , 171
2	$w_1 = 0.1$, $w_2 = 0.9$	193 , 173
3	$w_1 = 0.2$, $w_2 = 0.8$	189 , 175
4	$w_1 = 0.3$, $w_2 = 0.7$	184 , 177
5	$w_1 = 0.4$, $w_2 = 0.6$	177 , 178
6	$w_1 = 0.5$, $w_2 = 0.5$	175 , 181
7	$w_1 = 0.6$, $w_2 = 0.4$	174 , 182
8	$w_1 = 0.7$, $w_2 = 0.3$	163 , 199
9	$w_1 = 0.8$, $w_2 = 0.2$	158 , 201
10	$w_1 = 0.9$, $w_2 = 0.1$	156 , 204
11	$w_1 = 1.0$, $w_2 = 0.0$	155 , 206

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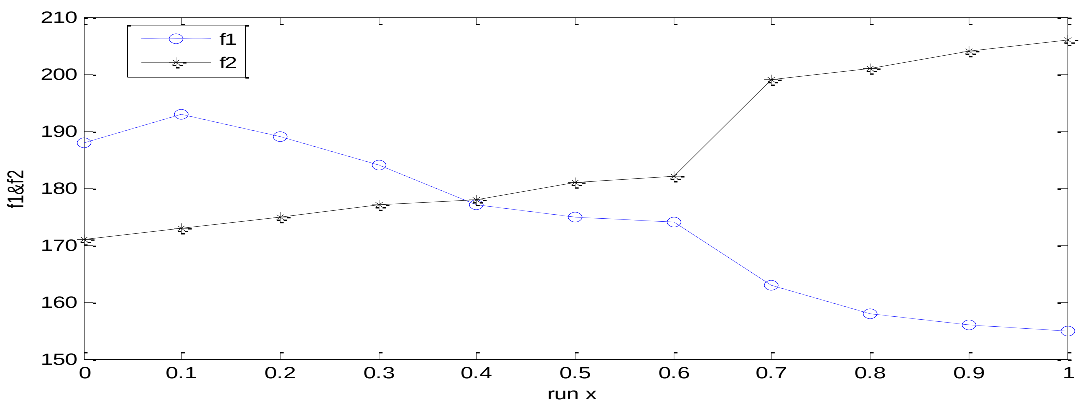
Table 2. Comparison between different approaches.

The name of approach	f_1	f_2
Interactive approach [24]	186	174
Fuzzy approach [2]	170	190
Fuzzy approach [7]	160	195
IFGP approach [3]	168	185
Trust Region Approach [6]	173	173
Proposed approach	177	178



Figuer 1: Plot showing the values of f_1 and f_2 as w_1 or w_2 changes linearly

4.3 Example 2

Consider the MOT problem mentioned in ([4] , [24]). To illustrate the application of the proposed algorithm. The problem has the following characteristics:

$$c^1 = \begin{bmatrix} 9 & 12 & 9 & 6 & 9 \\ 7 & 3 & 7 & 7 & 5 \\ 6 & 5 & 9 & 11 & 3 \\ 6 & 8 & 11 & 2 & 2 \end{bmatrix}, \quad c^2 = \begin{bmatrix} 2 & 9 & 8 & 1 & 4 \\ 1 & 9 & 9 & 5 & 2 \\ 8 & 1 & 8 & 4 & 5 \\ 2 & 8 & 6 & 9 & 8 \end{bmatrix},$$

and

$$c^3 = \begin{bmatrix} 2 & 4 & 6 & 3 & 6 \\ 4 & 8 & 4 & 9 & 2 \\ 5 & 3 & 5 & 3 & 6 \\ 6 & 9 & 6 & 3 & 1 \end{bmatrix}$$

with *Supplies* $a_1 = 5$, $a_2 = 4$, $a_3 = 2$, and $a_4 = 9$ and demands $b_1 = 4$, $b_2 = 4$, $b_3 = 6$, $b_4 = 2$, and $b_5 = 4$. The mathematical programming model of the above problem is written as follows:

$$\begin{aligned} minimize\ \ f_1 = {} & 9x_{11} + 12x_{12} + 9x_{13} + 6x_{14} + 9x_{15} \\ & + 7x_{21} + 3x_{22} + 7x_{23} + 7x_{24} + 5x_{25} \\ & + 6x_{31} + 5x_{32} + 9x_{33} + 11x_{34} + 3x_{35} \\ & + 6x_{41} + 8x_{42} + 11x_{43} + 2x_{44} + 2x_{45}, \end{aligned}$$

$$\begin{aligned} minimize\ \ f_2 = {} & 2x_{11} + 9x_{12} + 8x_{13} + x_{14} + 4x_{15} \\ & + x_{21} + 9x_{22} + 9x_{23} + 5x_{24} + 2x_{25} \\ & + 8x_{31} + x_{32} + 8x_{33} + 4x_{34} + 5x_{35} \\ & + 2x_{41} + 8x_{42} + 6x_{43} + 9x_{44} + 8x_{45}, \end{aligned}$$

$$minimize\ \ f_3 = 2x_{11} + 4x_{12} + 6x_{13} + 3x_{14} + 6x_{15}$$

$$+4x_{21} + 8x_{22} + 4x_{23} + 9x_{24} + 2x_{25}$$
$$+5x_{31} + 3x_{32} + 5x_{33} + 3x_{34} + 6x_{35}$$
$$+6x_{41} + 9x_{42} + 6x_{43} + 3x_{44} + x_{45},$$
$$subject\ to \quad x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 5,$$
$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 4,$$
$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 2,$$
$$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} = 9,$$
$$x_{11} + x_{21} + x_{31} + x_{41} = 4,$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 4,$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 6,$$
$$x_{14} + x_{24} + x_{34} + x_{44} = 2,$$
$$x_{15} + x_{25} + x_{35} + x_{45} = 4,$$
$$x_{ij} \geq 0, \quad i = 1,2,3,4 \quad j = 1,2,3,4,5.$$

4.2.2 Results and Discussions of Example 2:

A weighting approach is used together with the active-set line search algorithm (2) to solve the above problem. As one weight is changed linearly in each case, the other two weights are generated randomly, such that

$\sum_{k=1}^{3} w_k = 1$ and $w_k \geq 0$ for all $k = 1,2,3$. The values of the weights which are used for three cases are illustrated in three tables (3-5).

Figures (2-4), show the objective functions obtained from six solutions corresponding to the six weights compared to the weights for three cases. We observe that the best compromise solutions are $f_1 = 130$, $f_2 = 105$ and $f_3 = 76$ which are occur at w_1 is 0.8000, w_2 is 0.1759 and w_3 is 0.0241.

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To evaluate the performance of the suggested approach we compare the best results with the recently reported methods are given in Table 6.

Table 3. Different weights (w_1 is changed linearly)

Run	w_1	w_2	w_3
1	0.0000	0.5721	0.4279
2	0.2000	0.6205	0.1795
3	0.4000	0.2118	0.3882
4	0.6000	0.1636	0.2364
5	0.8000	0.1759	0.0241
6	1.0000	0.0000	0.0000

Table 4. Different weights (w_2 is changed linearly)

Run	w_1	w_2	w_3
1	0.6028	0.0000	0.3972
2	0.5676	0.2000	0.2324
3	0.4573	0.4000	0.1427
4	0.2718	0.6000	0.1282
5	0.1468	0.8000	0.0532
6	0.0000	1.0000	0.0000

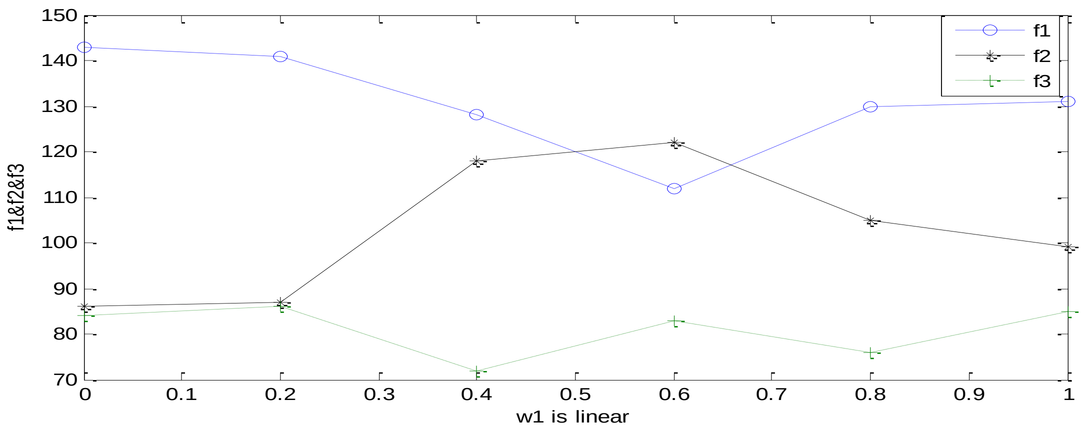
Table 5. Different weights (w_3 is changed linearly)

Run	w_1	w_2	w_3
1	0.7477	0.2523	0.0000
2	0.5994	0.2006	0.2000

3	0.4576	0.1424	0.4000
4	0.1354	0.2646	0.6000
5	0.1076	0.0924	0.8000
6	0.0000	0.0000	1.0000

Table 6. Comparison between different approaches.

The name of approach	f_1	f_2	f_3
Fuzzy approach [2]	112	106	80
Interactive approach [3]	127	104	76
Trust Region Approach [6]	144	104	73
Product approach [5]	157	72	86
Proposed approach	130	105	76



Figuer 2: Plot showing the values of f_1, $f_2$and f_3 solution for different weights in 6 runs of table 3.

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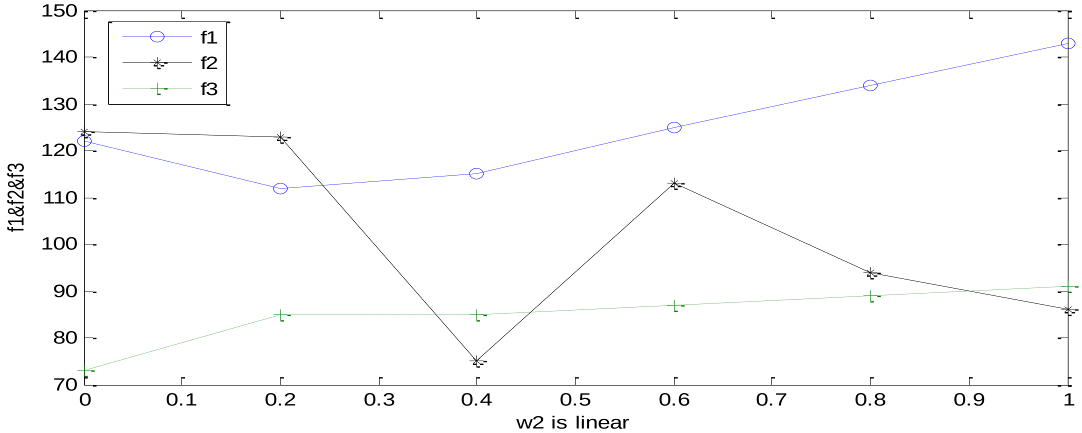
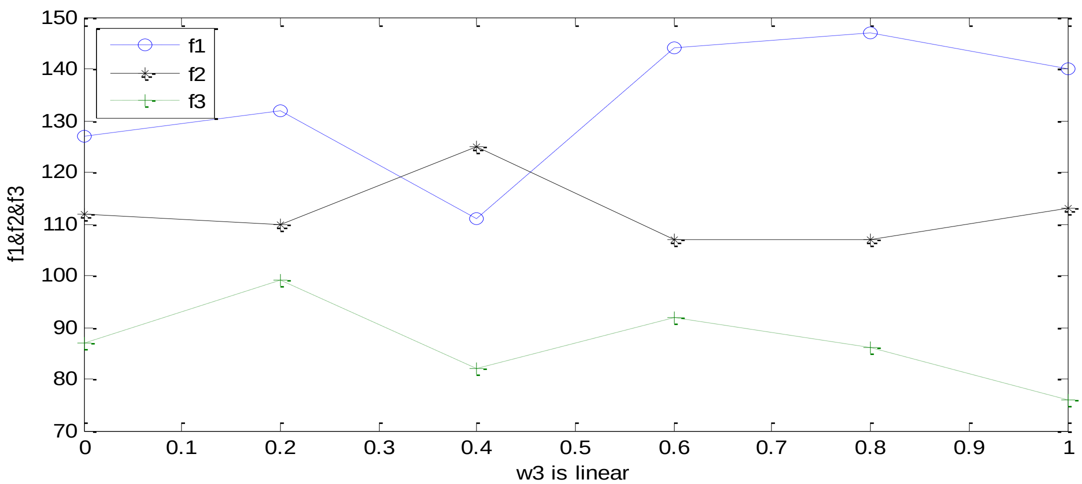


Figuer 3: Plot showing the values of f_1, $f_2$and f_3 solution for different weights in 6 runs of table 4.



Figuer 4: Plot showing the values of f_1, $f_2$and f_3 solution for different weights in 6 runs of table 5.

5 Concluding Remarks

In this paper an algorithm is proposed for a MOT problem and this approach is used to find the compromise solution of MOT problem. A weighting approach is used together with an active set strategy and a multiplier method to transform MOT problem to unconstrained optimization problem and we used an active-set line-search algorithm to solve it.

Two Numerical examples are illustrated and obtained results compared with some of the methods in literature. The comparison shows that the compromise solution is better and acceptable in real life situation when more than one objective available in transporting a product.

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