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Investigation Into Accuracy Of LGD2006 For Medium-Elevation Areas

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ABSTRACT

This paper presents a study about the Libyan Geodetic Datum 2006 (LGD2006) where a triangulation network has been established in a medium-elevation area in Libya. The network is consisted of braced quadrilaterals of 45 km lines in direction of meridians and 7 km lines in direction of parallels. The network distances and angles were accurately measured and then the coordinates were computed. In addition, coordinates of the major traverses points were measured using static GPS observation technique for several hours to ensure the maximum accuracy. GPS measurements were conducted using the World Geodetic System of 1984 (WGS84). Inverse geodetic methods were used to compare the achieved results with those of the Libyan ordinance survey. The results show that the best fit datum for medium-elevation areas (300-500m) in Libya is LGD2006 whereas WGS84 is best for low-elevation areas.

Keywords: LGD2006, triangulation, static GPS, accuracy, inverse geodetic problems.

1 Introduction

1.1 Background

As the surface of the earth is irregular and complex, for many centuries geodesists tried to determine the shape of the earth. They found that the most complex model of the earth is the geoid and the simplest model is the ellipsoid. This has led to many different reference ellipsoids around the world. Each country takes the newest reference ellipsoid as its reference datum for surveying purposes. Using an incorrect datum to express coordinates can result in position errors of hundreds of meters. As a result, countries modify the global datums to best fit their topographic relief by minimising the geoid undulations. The resulted new datum is known as the local datum [7].

European datum of 1950 (ED1950) had been used in Libya in late fifties and early sixties of last century. This datum best fits Europe not North Africa. European-Libyan datum of 1979 (ELD1979) is another datum that has been used in Libya. Nowadays the Libyan Geodetic Datum of 2006 (LGD2006), which is based on the international ellipsoid of 1924, is the most used datum for surveying applications in Libya [5].

Several researches found that the used ellipsoid for Libya fits only the northern part of the country because of deformations in the datum and that the used ground control points had been established by different companies using different measuring methods.

1.2 Aim and Objectives

The overall aim of this paper is to investigate the accuracy of using the LGD2006 for the medium-elevation areas in Libya. This aim will be assessed through investigating the following objectives:

- Choosing study area where ordinance survey ground control points are available.
- Establishment of a triangulation network of braced quadrilaterals of 45 km lines in direction of meridians and 7 km lines in direction of parallels.
- Using static GPS to measure the coordinates of Laplace station.
- Using geodetic formulas to compute distances and comparing results with those obtained by field observations.
- Establishment of new accurate ground control points to be used in other surveying applications.

2 Test site and Apparatus Used

2.1 Test Site

The test site is located in a medium-elevation area (300 to 400m above mean sea level) close to the city of Tarhuna. The site is between longitudes of 13° 30′ E and 13° 54′ E and latitudes of 31° 21′ N and 31° 52′ N. The test site is shown in Figure 1 while Figure 2 illustrates the topography of the site.

This site was chosen according to its elevation and to availability of two ground control points from the ordinance survey of Libya. These control points are called GPS 12-3 and GPS 12-4 and are located close to the site. Their coordinates are in LGD2006.

2.2 Apparatus Used

The instruments used in this research are:

- 1″ Total station from Trimple
- Prism
- Leica dual frequency GPS receiver
- Communication equipment

3. Trials, Results and Analysis

3.1 Observation Techniques

A total number of two accurate ground control points were available from the ordinance

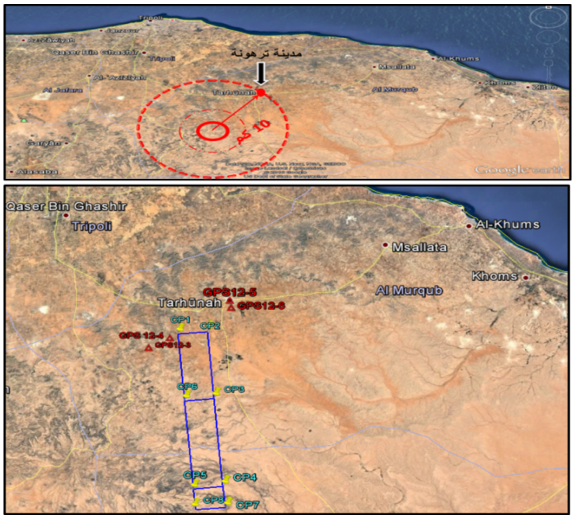

Figure 1: *Test site (source: Google earth)*

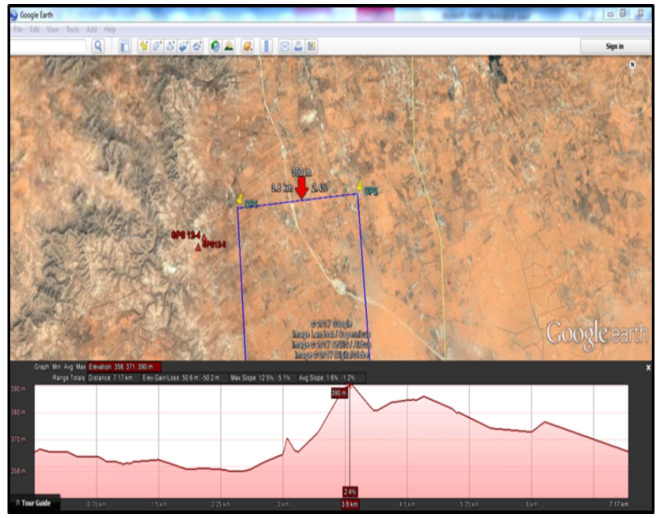

Figure 2: *Topography of the test site*

survey of Libya. These points were collected using static GPS with an estimated accuracy of 0.0068 m. These points were used as starting points to establish the network's other points. The number of established control points is 8 and were called CP_1 to CP_8 . In-between, points were called X_1, X_2, ……………., X_n . Figure 3 depicts the established control points.

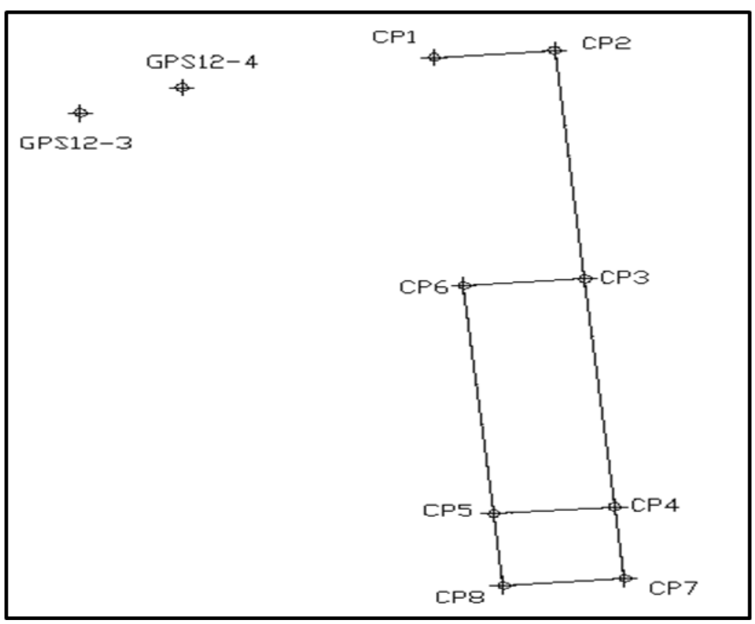

Figure 3: *Established control points*

The lines' length of the established triangulation network is 45km in the direction of meridians and 7km in the direction of parallels. These lengths were chosen so that the curvature of the earth will be taken into account. The chosen shape of the network is the braced quadrilateral. The control point GPS 12-4 was occupied by the total station and back sight was taken on GPS 12-3 to determine the true north. Then using the total station, the coordinates of all points, azimuths of lines and lengths of network lines were computed. In addition, static GPS observation technique, for several hours, was used to get the coordinates of the control points in WGS84.

3.2 Results and Discussion

The obtained coordinates for the control points in LGD2006 (UTM6º) and in WGS84 are shown in Table 1.

Table 1: *coordinates of control points*

Coordinates of network points in LGD2006 (UTM6º)			Coordinates of network points in WGS84		
point Id	Easting (m)	Northing (m)	Latitude (φ)	Longitude (λ)	Ellip.Hgt (m)
GPS12-3	356678.648	3578390.064	32° 19' 57.82" N	13° 28' 38.18" E	410.66
GPS12-4	357214.163	3578344.043	32° 19' 56.58" N	13° 28' 58.68" E	402.41
CP1	359198.744	3579582.980	32° 20' 38.92" N	13° 30' 11.68" E	400.12
CP2	366328.401	3580205.442	32° 21' 02.28" N	13° 34' 44.03" E	402.12
CP3	368067.873	3560281.212	32° 10' 16.13" N	13° 36' 00.50" E	418.49
CP4	369830.714	3540088.677	31° 59' 21.40" N	13° 37' 17.63" E	468.53
CP5	362699.684	3539466.102	31° 58' 58.15" N	13° 32' 46.33" E	541.94
CP6	360936.841	3559658.352	32° 09' 52.84" N	13° 31' 28.68" E	488.91
CP7	370308.386	3534617.252	31° 56' 22.78" N	13° 37' 40.67" E	492.14
CP8	363177.398	3533994.688	31° 55' 59.56" N	13° 33' 9.52" E	489.86

3.2.1 Transformation of Cartesian coordinates to geodetic coordinates

As the used coordinate system in the network is the Cartesian system and the ellipsoid represents the geodetic coordinate system, it's necessary to represent the relationship between the two systems mathematically according to the theory of cylindrical conformal projection as follows [6]:

$$\varphi = f(X, Y) \quad , \quad \lambda = f(X, Y) \qquad , \qquad X = f(\varphi, \lambda) \quad , \quad Y = f(\varphi, \lambda)$$

Conformal projection has two projection parameters, forward and reverse.

1- Forward (direct) parameters to get Cartesian coordinates

The assumptions put by Kruger to solve the projection equation are to keep the longitudes constant function and to make latitudes variable function as follows:

$$X = X_0 + C_2\lambda^2 + C_4\lambda^4 + C_6\lambda^6 + C_8\lambda^8 + C_{10}\lambda^{10} + \cdots$$

$$y = C_1\lambda + C_3\lambda^3 + C_5\lambda^5 + C_7\lambda^7 + C_9\lambda^9 + C_{11}\lambda^{11} + \cdots$$

2- Reverse (inverse) parameters to get geodetic coordinates

$$\varphi = B_x + C'_2 y^2 + C'_4 y^4 + C'_6 y^6 + C'_8 y^8 + \cdots$$
$$\lambda = C'_1 y + C'_3 y^3 + C'_5 y^5 + C'_7 y^7 + C'_9 y^9 + \cdots$$

where $C_1, C_2, C_3, C_4, C_5, \ldots$ are the forward parameters and

$C'_1, C'_2, C_{\prime 3}, C'_4, C'_5, \ldots$ are the reverse parameters

In addition, Bidshivalf theory was used to get the reverse parameters as following [4]:

$P'_0 = 1 \quad , \quad Q'_0 = 0$

$P'_1 = dx \quad , \quad Q'_1 = y$

$P'_2 = P'^2_1 - Q'^2_1 \quad , \quad Q'_2 = 2P'_1\, Q'_1$

$P'_3 = P'_1 P'_2 - Q'_1 Q'_2 \quad , \quad Q'_3 = P'_1\, Q'_2 + Q'_1 P'_2$

$P'_n = P'_1 P'_{n-1} - Q'_1 Q'_{n-1} \quad , \quad Q'_n = P'_1 Q'_{n-1} + Q'_1 P'_{n-1}$

The values of $P'_1, P'_2, P'_3, \ldots P'_n$ are substituted back into the following formula to transform Cartesian coordinates to geodetic ones:

$$q = q_0 + \sum_{j=1}^{n} C'_j P'_j \quad \rightarrow \quad q = q_0 + C'_1 P'_1 + C'_2 P'_2 + C'_3 P'_3 + \cdots$$

$$L = L_0 + \sum_{j=1}^{n} C'_j Q'_j \quad \rightarrow \quad L = l_0 + C'_1 Q'_1 + C'_2 Q'_2 + C'_3 Q'_3 + \cdots$$

$$q_0 = ln\sqrt{\left[\frac{1 + sin\varphi}{1 - sin\varphi}\right]\left[\frac{1 - e * sin\varphi}{1 + e * sin\varphi}\right]^e}$$

Latitude can be obtained from the following new formula developed by the geodesist Bidshivalf

$$B = 2\, arctan\left[\sqrt{\left[\frac{1 - esinB_0}{1 + esinB_0}\right]^e} . Exp(q)\right] - \frac{\pi}{2}$$

Where C' represents the reverse parameters, L represents the longitude, B is the latitude and B_0 is the central latitude.

The process is iterative so a Matlab program was developed to perform the conversion of coordinates from Cartesian to geodetic and the reverse process with the possibility to change datums as required. The program has been tested using the ordinance survey control point (GPS 12-3), the discrepancies in coordinates were close to zero and they were as a result of using 8 parameters in the program while the ordinance survey used only 5 parameters.

The Matlab program was used to convert the LGD2006 (UTM6°, International Hayford1924) network coordinates to geodetic coordinates, the results are shown in Table 2.

Table 2: *Network Cartesian and geodetic coordinates in LGD2006 (UTN6º)*

point Id	**Longitude (λ)**	**Latitude (φ)**	**Easting (m)**	**Northing (m)**
GPS12-3	13.47727320 º	32.33272965 º	356678.648	3578390.064
GPS12-4	13.48296802 º	32.33238317 º	357214.163	3578344.043
Control Point 1	13.50386317 º	32.34380816 º	359198.782	3579582.981
Control Point 2	13.57951374 º	32.35029741 º	366328.427	3580205.404
Control Point 3	13.60075344 º	32.17081670 º	368067.856	3560281.184
Control Point 4	13.62217705 º	31.98894569 º	369830.460	3540092.809
Control Point 5	13.54681925 º	31.98249019 º	362700.774	3539470.353
Control Point 6	13.52524994 º	32.16434433 º	360938.280	3559658.728
Control Point 7	13.55264639 º	31.93321194 º	370307.987	3534622.442
Control Point 8	13.62796417 º	31.93966278 º	363178.325	3533999.995

3.2.2 comparing obtained results and ordinance survey results using geodetic formulas

The reverse geodetic formulas were used to compute the lengths of network lines to compare them to those of the ordinance survey to determine the accuracy of LGD2006 in medium-elevation areas.

To compute the distance between two points, first a number of parameters has to be calculated as following [6]:

$$e' = \sqrt{\frac{a^2 - b^2}{b^2}} \quad , \quad e = \sqrt{\frac{a^2 - b^2}{a^2}} \quad , \quad C = \frac{a^2}{b} \quad , \quad \rho = \frac{pi}{180} \quad , \quad fm = \frac{\varphi_1 + \varphi_2}{2 * \rho} \quad ,$$

$$ff = \frac{\varphi_2 - \varphi_1}{\rho} \quad , \quad Lm = \frac{\lambda_2 - \lambda_1}{\rho}$$

$$W_1 = \sqrt{(1 - e^2 \sin^2 \varphi_1)} \quad , \quad W_2 = \sqrt{(1 - e^2 \sin^2 \varphi_2)} \quad , \quad N = \frac{C}{\sqrt{(1 + H)}} \quad ,$$

$$M = \frac{N}{1 + H} \quad , \quad H = e'^2 * COS^2(fm)$$

$$\sin u_1 = \frac{\sin \varphi_1 . \sqrt{1 - e^2}}{W_1} \quad , \quad \sin u_2 = \frac{\sin \varphi_2 \sqrt{1 - e^2}}{W_2} \quad \cos u_1 = \frac{\cos \varphi_1}{W_1} \quad ,$$

$$\cos u_2 = \frac{\cos \varphi_2}{W_2}$$

$$P = \cos u_2 . \sin w \quad , \quad q = \cos u_1 . \sin u_2 - \sin u_1 . \cos u_2 \cos w$$

$$\cot x_1 = \cot u_2 . \cos \Delta w \quad , \quad \cot x_2 = \cot u_1 . \cos \Delta w$$

$$\tan \Delta\beta_{12} = \frac{\tan q}{\tan p} = \frac{\cos x_1 . \tan \Delta w}{\sin(x_1 - u_1)} \, , \, \beta_{21} = \frac{\tan q_1}{\tan p_1} = -\frac{\cos x_2 . \tan \Delta w}{\sin (x_2 - u_2)}$$

$$\beta_{21} = \beta_{12} \pm 180 + \Delta\beta$$

Using the above parameters, distances can be computed using two methods, one for short distances and the other is for long ones.

1- Geodetic formulas for short distances

$$Z = Lm\,.\,N\,.\,COS(fm)\left(1 + (1 - 9.e'^2 + 8.H^2).\frac{ff^2}{24} - \frac{(Lm\,.\,Sin(fm))^2}{24}\right)$$

$$Q = ff.M\left(1 - \left(e'^2 - 2H^2\right).\frac{b^2}{8} - \frac{(1 + H^2)\ (Lm.COS(fm))^2}{12} - \frac{(Lm.COS(fm))^2}{8}\right)$$

$$S = \sqrt{Z^2 + Q^2}$$

2- Geodetic formulas for long distances

If the azimuth is less than 45°, the formula used is: $\tan\sigma = \frac{\tan p}{\cos\beta_{12}}$

If the azimuth is more than 45°, the formula is: $\sin\sigma = \frac{\sin q_2}{\sin\beta_{12}} = \frac{\cos u_2 . \sin\Delta w}{\sin\beta_{12}}$

Where:

e, e': first and second eccentricity

u_1, u_2: reduced latitude of the point

β_{12}, β_{21}: forward and back azimuth

Δw: difference between longitudes

S: measured short distance on ellipsoid

M: meridional radius of curvature

N: radius of curvature in the prime vertical

$\Delta\beta$: convergence of meridians

p , q: lines lengths from spherical triangle

$\boldsymbol{\sigma}$: measured long distance on ellipsoid

Another Matlab program was developed using the above formulas to solve the reverse geodetic problems of short and long distances using LGD2006. The results are presented in Table 3.

Table 3: *Distances between network points using LGD2006*

Distance	points	$\boldsymbol{\varphi}$	$\boldsymbol{\lambda}$	Distance measured by total station	Reverse formulas for short distances	Reverse formulas for long distances
S_{6-3}	Point(cp6)	32.16434433	13.525249944	7158.019	7157.938	7157.939
	Point(cp3)	32.17081670	13.600753444			
S_{3-4}	Point(cp3)	32.17081670	13.600753444	20269.338	20268.990	20268.991
	Point(cp4)	31.98894569	13.622177055			
S_{4-5}	Point(cp4)	31.98894569	13.622177055	7158.155	7158.091	7158.092
	Point(cp5)	31.98249019	13.546819250			
S_{5-6}	Point(cp5)	31.98249019	13.546819250	20269.055	20268.500	20268.502
	Point(cp6)	32.16434433	13.525249944			
S_{4-7}	Point(cp4)	31.98894569	13.622177055	5492.236	5492.220	5492.220
	Point(cp7)	31.93966292	13.627964358			
S_{7-8}	Point(cp7)	31.93966292	13.627964358	7158.112	7158.076	7158.077
	Point(cp8)	31.93321203	13.552646394			
S_{8-5}	Point(cp8)	31.93321203	13.552646394	5492.129	5492.086	5492.086
	Point(cp5)	31.98249019	13.546819250			

Using the same Matlab program the distances were computed using the WGS84 datum, the results are shown in Table 4.

Table 4: *Distances between network points using WGS84*

Distance	points	φ	λ	Distance measured by total station	Reverse formulas for short distances	Reverse formulas for long distances
S_{6-3}	Point(cp6)	32.16467923	13.52463410	7158.019	7157.908	7157.909
	Point(cp3)	32.17115206	13.60014078			
S_{3-4}	Point(cp3)	32.17115206	13.60014078	20269.338	20268.888	20268.889
	Point(cp4)	31.98927792	13.62156647			
S_{4-5}	Point(cp4)	31.98927792	13.62156647	7158.155	7158.056	7158.057
	Point(cp5)	31.98282194	13.54620554			
S_{5-6}	Point(cp5)	31.98282194	13.54620554	20269.055	20268.401	20268.402
	Point(cp6)	32.16467923	13.52463410			
S_{4-7}	Point(cp4)	31.98927792	13.62156647	5492.236	5492.195	5492.195
	Point(cp7)	31.93999427	13.62735433			
S_{7-8}	Point(cp7)	31.93999427	13.62735433	7158.112	7158.042	7158.043
	Point(cp8)	31.93354290	13.55203324			
S_{8-5}	Point(cp8)	31.93354290	13.55203324	5492.129	5492.062	5492.062
	Point(cp5)	31.98282194	13.54620554			

3.3 Discussion of results

The field measurements by the total station is considered as a reference for the purpose of comparison. Comparing the field observations by the total station using the reverse geodetic problems, ordinance survey measurements using LGD2006, and GPS measurements using WGS84, it's clear from Table 3 and Table 4 that there are some differences between the above mentioned measurements. The distances using LGD2006 are closer to reference distances. From Table 3, the biggest difference between reference distances and short distances computed by reverse geodetic problems is 55cm which is between control points 5 and 6 and the smallest difference for short distances is 16mm which is between control points 4 and 7. For long distances using LGD2006, the results are almost the same for short distances. When WGS84 datum was used to compute the distances, either short or long, using geodetic problems (Table 4), the differences between the resulted distances and the reference ones are bigger than those obtained when using LGD2006 datum. The biggest difference is 65cm between control points 5 and 6 and the smallest difference is 41mm between control points 4 and 7.

4 Conclusion and recommendations

A triangulation network was established on a large lot of medium-elevation land using UTM6º (zone 33) and a number of field and office trials and tests were conducted to check the accuracy of LGD2006 in medium-elevation areas. The results show that the best fit datum for areas of elevations 300 to 500m is the LGD2006 and WGS84 is best for areas of low elevations.

Based on the research findings, the following recommendations are being made for possible future work.

- Another study should be made on high-elevation areas to check the suitability of LGD2006 in those areas.
- Continuing studies on the ellipsoid used in Libyan datum and its conformance with the geoid.
- Using static GPS to establish high accuracy first class ground control point in all regions of Libya.

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