مجـلة الـتربـوي
Journal of Educational
ISSN: 2011- 421X
Arcif Q3

معامل التأثير العربي 1.5
العدد 21

مجلة التربوي
مجلة علمية محكمة تصدر عن كلية التربية
جامعة المرقب

العدد الحادي والعشرون
يوليو 2022م

هيئـــة التحريـر

رئيس هيئة التحرير:	د. مصطفى المهدي القط
مدير التحرير:	د. عطية رمضان الكيلاني
سكرتير المجلة:	أ. سالم مصطفى الديب

- المجلة ترحب بما يرد عليها من أبحاث وعلى استعداد لنشرها بعد التحكيم .
- المجلة تحترم كل الاحترام آراء المحكمين وتعمل بمقتضاها .
- كافة الآراء والأفكار المنشورة تعبر عن آراء أصحابها ولا تتحمل المجلة تبعاتها .
- يتحمل الباحث مسؤولية الأمانة العلمية وهو المسؤول عما ينشر له .
- البحوث المقدمة للنشر لا ترد لأصحابها نشرت أو لم تنشر .

(حقوق الطبع محفوظة للكلية)

مجـلة الـتربـوي
Journal of Educational
ISSN: 2011- 421X
Arcif Q3

معامل التأثير العربي 1.5
العدد 21

ضوابط النشر:

يشترط في البحوث العلمية المقدمة للنشر أن يراعى فيها ما يأتي :

- أصول البحث العلمي وقواعده .
- ألا تكون المادة العلمية قد سبق نشرها أو كانت جزءا من رسالة علمية .
- يرفق بالبحث تزكية لغوية وفق أنموذج معد .
- تعدل البحوث المقبولة وتصحح وفق ما يراه المحكمون .
- التزام الباحث بالضوابط التي وضعتها المجلة من عدد الصفحات ، ونوع الخط ورقمه ، والفترات الزمنية الممنوحة للتعديل ، وما يستجد من ضوابط تضعها المجلة مستقبلا .

تنبيهات :

- للمجلة الحق في تعديل البحث أو طلب تعديله أو رفضه .
- يخضع البحث في النشر لأولويات المجلة وسياستها .
- البحوث المنشورة تعبر عن وجهة نظر أصحابها ، ولا تعبر عن وجهة نظر المجلة .

Information for authors

1- Authors of the articles being accepted are required to respect the regulations and the rules of the scientific research.
2- The research articles or manuscripts should be original and have not been published previously. Materials that are currently being considered by another journal or is a part of scientific dissertation are requested not to be submitted.
3- The research articles should be approved by a linguistic reviewer.
4- All research articles in the journal undergo rigorous peer review based on initial editor screening.
5- All authors are requested to follow the regulations of publication in the template paper prepared by the editorial board of the journal.

Attention

1- The editor reserves the right to make any necessary changes in the papers, or request the author to do so, or reject the paper submitted.
2- The research articles undergo to the policy of the editorial board regarding the priority of publication.
3- The published articles represent only the authors' viewpoints.

مجـلة الـتربـوي
Journal of Educational
ISSN: 2011- 421X
Arcif Q3

معامل التأثير العربي 1.5
العدد 21

Some applications of harmonic functions

Abdulbasit Alzubayr Abdulrahman[1], Omar Ismael Elhasadi[2], Zaynab Ahmed Khalleefah[3]
[1]Department of Mathematics,Sebha University, Obari, Libya
[2]Department of Mathematical Sciences, Libyan Academy, Tripoli, Libya.
[3]Department of Mathematics, Garyan University, Garyan, Libya
Zaynab.Zuwaliyah@gu.edu.ly[3]

Abstract
In this paper, we introduce the most important applications of harmonic functions, and the most important related to harmonic functions.
We have studied the Dirichlet problem related to harmonic functions.
We also studied the most important applications of the Laplace field, including the development of the magnetic phase and temperatures, we obtained results that are more accurate, including the use of Laplace field reduces noise as well as the use of the harmonic functions in measuring magnetic susceptibility leads to the correction of the contact shift.
Keywords: harmonic functions, Laplace field, Dirichlet problem.

1. Introduction
The harmonic functions have many applications including temperature and conserved potentials in physical sciences.
It is also valid to extend the harmonic function concept and basic properties to vectors such as electrostatic field magneto static field current density flow velocity and gravitational force we identify them as Laplace fields these important physical fields are widely applied in areas such as physics chemistry, biology, engineering and medicine. Precise measurements of these fields and related physical quantities such as the magnetic susceptibility are important.
In this paper, velop the Laplace field theory and methods applicable to high precision measurement of these physical fields and quantitation of related physical quantities such as magnetic susceptibility for objects of arbitrary shapes.

2. Applications
2.1 High Precision Field Mapping and Temperature Mapping
The mean value property of harmonic function fields is applied for reduction of random noise in the measurement of these fields. With magnetic phase imaging,

three-dimensional field maps are obtained with high precision, i.e. 10^{-11}T for magnetic fields and 10^{-3}°C for the temperature.
Many physical quantities such as temperature, electric field, electric potential, and magnetic field belong to Laplace fields in space under certain conditions.
The mean value property, one of the common properties of Laplace field, states that the mean field value on a spherical surface or volume Ω is equal to the field value at the center of that region, i.e.

$$u(R) =< u(R,r) > \Omega \qquad (1)$$

In which R is the position vector of the censer, r is the vector eriginaned at R and spanning the whole space of (Eq 1 & Fig 1). The application of Eq (1) the spherical mean value analysis can effectively reduce the noise level of the physical quantity measurements with data on a sphere. The average of data on the sphere decreases the random noise of the center value by a factor of$\sqrt{N}$, with N a total member of the data points for averaging in a three-dimensional image data set.
A sphere of radius r has about $N = \frac{4\pi}{3}r^3$ pixels. When e is fixed, the central value, replaced by the average of all the pixels in the sphere, has its random noise reduced at most by a factor of $\sqrt{\frac{4\pi}{3}}r^{3/2}$. With the radius of typical image size 100 pixels, this is a noise reduction by 2046 times.
Both static magnetic field and phase maps are Laplace fields in the region with homogeneous magnetic susceptibility, and they exhibit the mean-value property of Laplace field. i.e.

$$B(R) =< B(R,r) > \Omega \qquad (2)$$

$$\emptyset(R) =< \emptyset(R,r) > \Omega \qquad (3)$$

We employ the phase map from MRPI as the basis for testing the noise reduction method.
We can effectively reduce the noise level of the MRPI phase maps to improve the precision of magnetic field and temperature mapping.
The measurement of magnetic field provides useful information for magnetic material study, earthquake prediction, and the study of free radicals, reaction rates and blood.
Components in magneto-chemistry. It also has potential for the study of human brain via magneto encephalography and the superconducting quantum

interference device (SQUID). In magnetic resonance spectroscopy and imaging, it is essential for precision field shimming, and very important for understanding the susceptibility effects related to the functional magnetic resonance imaging and tumor oxygenation studies in medicine. Temperature measurement broadly relates to physical sciences, physiology and medicine. In particular, with medical interventions such as thermotherapy, direct monitoring of temperature in tissues is essential for accurate assessment of treatment load and efficacy, and for understanding the physiological responses of tissues or organs. With medical interventions such as thermal-therapy, the direct monitoring of temperature in tissue is necessary for accurate evaluation of the treatment load and effectiveness, and understanding the tissue or organ physiological responses However, current MRPI technology has precision limits of about 10^{-8}Tesla (T) for magnetic field mapping and 0.5 °C for temperature mapping with proton signals (21). The Laplace field property we can generate field maps with high with precision, i.e. $10^{-11}{\sim}10^{-12}$T for magnetic field, $10^{-3}{\sim}10^{-4}$ for temperature.

The method can be easily generalized to applications for many other physical quantities whose distributions in space are Laplace fields. The improved precision for the measurements of those fields will hold great significance for the advancement of scientific studies.

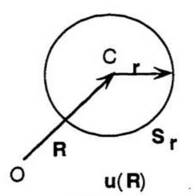

Fig1

2.2 The Measurements of Magnetic Susceptibility

Magnetic susceptibility measurement has wide-ranging applications, particularly for medical applications in vivo. A magnetic susceptibility quantitation method is developed for susceptibility objects of arbitrary shapes, without the presumption of homogeneous main field. Based on the Laplace field theory, the external magnetic field at the location of interest inside the susceptibility object can be exactly derived from the field values on a spherical surface (3D phantom) enclosing the object and centered at that location. With the numerical computation of the self-demagnetizing field and the correction of contact shift,

magnetic susceptibility maps are obtained for CuSO4 containers of various shapes immersed in water.
Substituting $M = xH$ into $B = \mu_0(H + M)$
$B = \mu_0(H + Hx)$
$B = \mu_0(1 + x)H = \mu_0\mu_r H$

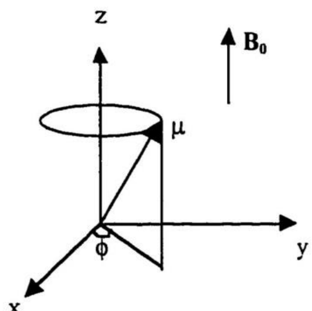

Fig2

Where x is the dimensionless volume magnetic susceptibility, $\mu_r = 1 + x$ is called relative magnetic permeability. The most common types of magnetism include diamagnetism $(x < 0)$ such as for magnetization of copper, Para magnetism $(x > 0)$ such as free radicals, and ferromagnetism $(x > 0)$ such as for magnetization of the iron.

2.3 Trajectory Generation

Let $\emptyset$ be a scalar potential function satisfying Laplace's equation, $\nabla^2\emptyset = 0$so that $\nabla\emptyset$ defines the streamlines of $\emptyset$. A simple streamline-following controller can be obtained by using the gradient of the harmonic function as a velocity or displacement for the effector.
$q = k_\nabla \nabla\emptyset$
Where k_∇ is the velocity gain?
This form of control has been implemented for a P-50 robot arm. Figure 3 shows an example of the arm using.

Fig3

A harmonic function to avoid a box placed in the center of its workspace. Five frames were superimposed in the figure to illustrate the robot's movement. The goal is a position forward and to the left of the box.

In figure 3, the starting configuration placed the end effector to the right and slightly behind the box. The harmonic function for the three-dimensional configuration space was computed from scratch in approximately 40 seconds using a Motorola 68030 processor. Once the harmonic function is computed, paths to the goal can be executed from any point in the workspace. The first three joints were used. The values for the joint angles over time are shown in Figure 5. This path was executed in approximately 5 seconds.

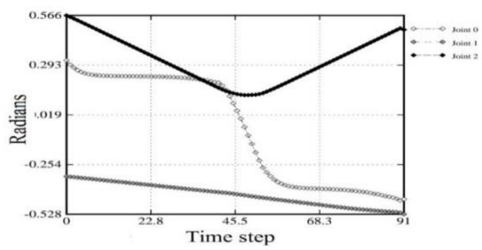

Fig4

2.4 Electrostatic Potential

In an electrostatic force field, the field intensity at a point is a vector positive charge placed at that point. The representing the force exerted on a unit electrostatic potential is a scalar function of the space coordinates such that, at each point, its directional derivative in any direction is the negative of the component of the field intensity in that direction.

For two stationary charged particles, the magnitude of the force of attraction or repulsion exerted by one particle on the other is directly proportional to the product of the charges and inversely proportional to the square of the distance between those particles. From this inverse-square law, it can be shown that the potential at a point due to a single particle in space is inversely proportional to the distance between the point and the particle. In any region free of charges, the potential due to a distribution of charges outside that region can be shown to satisfy Laplace’s equation for three-dimensional space.

If conditions are such that the potential V is the same in all planes parallel to the xy plane, then in regions free of charges V is a harmonic function of just the two variables x and y:

$$V_{xx}(x,y) + V_{yy}(x,y) = 0.$$

The field intensity vector at each point is parallel to the xy plane, with x and y components$-V_x(x,y)$and $-V_y(x,y)$respectively. That vector is, therefore, the negative of the gradient of $V(x,y)$.

A surface along which $V(x,y)$ is constant is an equipotential surface. The tangential component of the field intensity vector at a point on a conducting surface is zero in the static case since charges are free to move on such a surface. Hence $V(x,y)$ is constant along the surface of a conductor, and that surface is an equipotential.

If U is a harmonic conjugate of V, the curves $U(x,y) = c_2$ in the xy plane arecalled flux lines. When such a curve intersects an equipotential curve $V(x,y) = c_1$ at a point where the derivative of the analytic function $V(x,y) + iU(x,y)$ is not zero, the two curves are orthogonal at that point and the field intensity is tangent to the flux line there.

Boundary value problems for the potential V are the same mathematical problems as those for steady temperatures T; and, as in the case of steady temperatures, the methods of complex variables are limited to two-dimensional problems. The problem posed in (see Fig 5), for instance, can be interpreted as that of finding the two-dimensional electrostatic potential in the empty space

$$-\frac{\pi}{2} < x < \frac{\pi}{2}, y > 0$$

Bounded by the conducting planes $x = \pm\pi/2$and $y = 0$, insulated at their intersections, when the first two surfaces are kept at potential zero and the third at potential unity.

The potential in the steady flow of electricity in a conducting sheet lying in a plane is also a harmonic function at points free from sources and sinks. Gravitational potential is a further example of a harmonic function in physics.

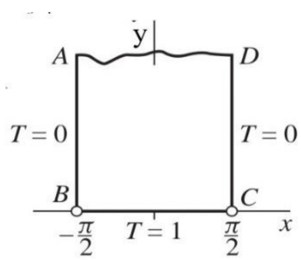

Fig5

2.5 Dirichlet problem in a quadrant

The boundary of a very large sheet of metal (thought of as the quarter plane Ω) is kept at the constant temperatures 100° on the bottom and 50° on the left, as illustrated in Figure 6. After a long enough period, the temperature inside the plate reaches an equilibrium distribution. Find this steady-state temperature $u(x, y)$.

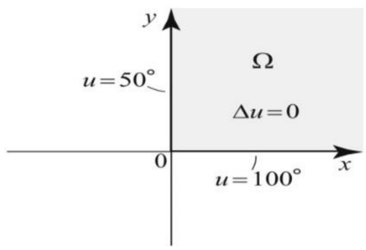

Fig6

The steady-state temperature is a solution of the Dirichlet problem, which consists of Laplace's equation $\Delta u = 0$, inside Ω; along with the boundary conditions

$u(x, 0) = 100,\ \ x > 0 u(0, y) = 50,\ \ y > 0$

Because the boundary data is independent of $r = |x + iy|$, we try for a solution of the harmonic function

$u(x, y) = a\ \ Arg(x + iy) + b$

Where a and b are real constants to be determined to satisfy the boundary conditions. From the first condition we obtain

$u(x, 0) = 100\ \Rightarrow\ a\, Arg\, x + b = 100\ \Rightarrow\ b = 100,$

As $Arg\ x = 0$ for $x > 0$. From the second condition

$u(0, y = 50\ \Rightarrow\ a\, Arg(iy) + b = 50$

$\Rightarrow a\frac{\pi}{2} + 100 = 50$

$\Rightarrow a = -\frac{100}{\pi}$

Since $Arg(iy) = \frac{\pi}{2}$ for $y > 0$, and b=100. Thus the steady-state temperature inside the plate is

$u(x, y) = -\frac{100}{\pi} Arg(z) + 100.$

Now for $z = x + iy$ with $x > 0$, we have

$Argz = tan^{-1}\left(\frac{y}{x}\right)$,

Therefore, another way of expressing the solution is

$$u(x.y) = -\frac{100}{\pi} tan^{-1}\left(\frac{y}{x}\right) + 100$$

The graph of u is shown in Figure 7. Note the temperature on the boundary; it matches the boundary conditions.

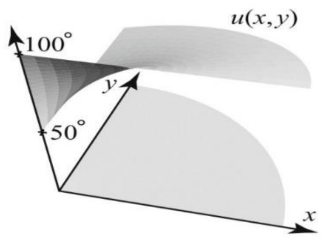

Fig7

2.6 Potential in a cylindrical space

A long hollow circular cylinder is made out of a thin sheet of conducting material, and the cylinder is split lengthwise to form two equal parts. Those parts are separated by slender strips of insulating material and are used as electrodes, one of which is grounded at potential zero and the other kept at a different fixed potential. We take the coordinate axes and units of length and potential difference as indicated on the left in Fig 8. We then interpret the electrostatic potential $V(x,y)$ over any cross section of the enclosed space that is distant from the ends of the cylinder as a harmonic function inside the circle $x^2 + y^2 = 1$in the xy plane. Note that $V = 0$ on the upper half of the circle and that $V = 1$ on the lower half.

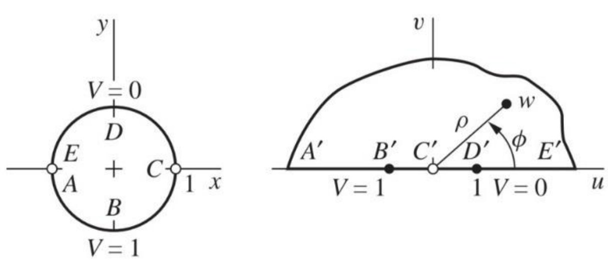

Fig8

A linear fractional transformation that maps the upper half plane onto the interior of the unit circle centered at the origin, the positive real axis onto the upper half of the circle, and the negative real axis onto the lower half of the circle.
Interchanging z and w there, we find that the inverse of the transformation

$$z = \frac{i-w}{i+w} \qquad (4)$$

Gives us a new problem for V in a half plane, indicated on the right in Fig8. Now the imaginary component of

$$\frac{1}{\pi} logw = \frac{1}{\pi} ln\rho + \frac{i}{\pi}\emptyset(\rho > 0)\,, 0 \le \emptyset \le \pi) \qquad (5)$$

Is a bounded function of u and v that assumes the two portions have the required constant values $\emptyset = 0$ and $\emptyset = \pi$ of the $u$axis. Hence the desired harmonic function for the half plane is

$$V = \frac{1}{\pi} \arctan\left(\frac{v}{u}\right) \qquad (6)$$

The arctangent function has values ranging from 0 to 1. The opposite of transformation (4) is

$$w = i\frac{1-z}{1+z} \qquad (7)$$

From which u and v can be expressed in terms of x and y. Equation (6) then becomes

$$V = \frac{1}{\pi} \arctan\left(\frac{1-x^2-y^2}{2y}\right)(0 \le \arctan t \le \pi) \qquad (8)$$

The function (8), because it is harmonic inside the circle and assumes the requisite values on the semicircles, is the potential function for the space contained by the cylindrical electrodes. We must keep in mind that if we want to validate this solution, we must keep in mind that
$lim_{t\to 0}\, arctant = 0\,, t > 0Andlim_{t\to 0}\, arctant = \pi, t < 0$
The equipotential curves $V(x,y) = c_1$ $(0 < c_1 < 1)$ in the circular region are arcs of the circles

$$x^2 + (y + tan\pi c_1)^2 = sex^2\pi c_1$$

Each circle should pass through the points $(\pm 1{,}0)$. In addition; the x-axis segment between those places is the equipotential $V(x,y) = 1/2$. A harmonic conjugate U of V is $-(1/\pi)$ in ρ, or the imaginary part of the function$-(i/\pi)Log\ w$. In view of equation (7), U may be written

$$U = -\frac{1}{\pi} ln \left|\frac{1-z}{1+z}\right|.$$

From this equation, we can see that the flux lines $U(x, y) = c_2$ are arcs of circles with centers on the x axis. The segment of the y axis between the electrodes is also a flux line.

3. Conclusions

From our study, we conclude the following:

- Using the technique of harmonic functions for temperature mapping gives results with an accuracy of about $10^{-3}°C$ Celsius, which in the past was about$0.5\ °C$, which helps in heat treatment through direct monitoring of temperatures in the tissues, which in turn gives an accurate evaluation of the treatment and its effectiveness and understanding. The extent of the response of tissues and organs with greater accuracy than previous measurements that do not use the technique of harmonic functions.
- Using the Laplace field over the MRPI field reduces noise by 2046 times and gives results that are more accurate.
- The use of harmonic functions in measuring magnetic susceptibility leads to the correction of the contact shift. CoSO4 containers of various shapes immersed in water as a model.
- The use of harmonic functions in calculating the electric potential gives a development to the method of complex variables from problems related to the study of the second dimension to finding the electric potential in empty space.

4. Recommendations

- We recommend generalizing the mean value property method of harmonic functions to the applications for the physical quantities whose distribution in space are Laplace fields because we believe that the improved accuracy will have a significant impact on the progress of scientific studies.
- Generalizing the idea of studying the magnetic field using the mean value property technique and using its accurate results to predict earthquakes and human brain and blood interaction rates through magnetic brain mapping.
- Studying the solution 00002of Dirichlet problems using the theorems of the Poisson integral equation and Fourier series, because it gives a development to the theory of harmonic functions and is more general.

5.References

1.Asmar, N. H., &Grafakos, L. (2018). *Complex Analysis with Applications*. Springer International Publishing.

2.Auchmuty, G., & Cho, M. (2015). Boundary integrals and approximations of harmonic functions. *Numerical Functional Analysis and Optimization*.

3. Berg, C. (2014). *Complex analysis*. Department of Mathematical Sciences, University of Copenhagen.

4. Bi, L. (2018). Internal and External Harmonic Functions in Flat-Ring Coordinates.

5. Brown, J. W., & Churchill, R. V. (2009). *Complex variables and applications*. Boston: McGraw-Hill Higher Education,

6. Cain, J. W., & Reynolds, A. M. (2010). Ordinary and Partial Differential Equations. *An Introduction to Dynamical Systems. Creative Commons. Virginia Commonwealth University*, *Richmond*.

7. Chasnov, J. R. (2014). Differential Equations with YouTube Examples-eBooks and textbooks from bookboon. Com.

8. Connolly, C. I., &Grupen, R. A. (1993). The applications of harmonic functions to robotics. *Journal of robotic Systems*.

9.Graczyk, J., &Świątek, G. (2019). Analytic structures and harmonic measure at bifurcation locus.

10. Guadie, M. A. (2013). Harmonic functions on square lattices: uniqueness sets and growth properties.

11. Hansen, W., &Nadirashvili, N. (1993). A converse to the mean value theorem for harmonic functions. ActaMathematica,

12. Karachik, V. V. (2007). On one representation of analytic functions by harmonic functions. *Mat. Tr*, *10*(2), 142-162.

13. Lundström, N. L. (2011). *P-harmonic functions near the boundary* (Doctoral dissertation, Umeåuniversitet, Institutionenförmatematikochmatematiskstatistik)

14. Mathews, J., & Howell, R. (2012). *Complex analysis for mathematics and engineering*. Jones & Bartlett Publishers.

15. McMullen, C.Bennewitz (2000). Complex Analysis. *Course Notes*, *Harward University Fall*, *2006*, 2010.

16. Rejeb, C. (2015). *Harmonic and subharmonic functions associated to root systems* (Doctoral dissertation).

17. Stavroulakis, I. P., &Tersian, S. A. (1999). *Partial differential equations: An introduction with Mathematica and MAPLE*. World Scientific Publishing Company.

18. Taylor, M. (2013). *Partial differential equations II: Qualitative studies of linear equations* (Vol. 116). Springer Science & Business Media.

	مجـلة الـتربـوي Journal of Educational ISSN: 2011- 421X Arcif Q3	معامل التأثير العربي 1.5 العدد 21

الفهـــــــــرس

مجـلة الـتربـوي
Journal of Educational
ISSN: 2011- 421X
Arcif Q3

معامل التأثير العربي 1.5
العدد 21