



مجلة التربوي
Journal of Educational
ISSN: 2011- 421X
Arcif Q3

معامل التأثير العربي 1.5
العدد 19



مجلة التربوي

مجلة علمية محكمة تصدر عن كلية التربية

جامعة المرقب

العدد التاسع عشر
يوليو 2021م

هيئة تحرير
مجلة التربوي

- المجلة ترحب بما يرد عليها من أبحاث وعلى استعداد لنشرها بعد التحكيم .
 - المجلة تحترم كل الاحترام آراء المحكمين وتعمل بمقتضاها .
 - كافة الآراء والأفكار المنشورة تعبر عن آراء أصحابها ولا تتحمل المجلة تبعاتها .
 - يتحمل الباحث مسؤولية الأمانة العلمية وهو المسؤول عما ينشر له .
 - البحوث المقدمة للنشر لا ترد لأصحابها نشرت أو لم تنشر .
- (حقوق الطبع محفوظة للكلية)



ضوابط النشر:

- يشترط في البحوث العلمية المقدمة للنشر أن يراعى فيها ما يأتي :
- أصول البحث العلمي وقواعده .
 - ألا تكون المادة العلمية قد سبق نشرها أو كانت جزءا من رسالة علمية .
 - يرفق بالبحث تزكية لغوية وفق أنموذج معد .
 - تعدل البحوث المقبولة وتصحح وفق ما يراه المحكمون .
 - التزام الباحث بالضوابط التي وضعتها المجلة من عدد الصفحات ، ونوع الخط ورقمه ، والفترات الزمنية الممنوحة للتعديل ، وما يستجد من ضوابط تضعها المجلة مستقبلا .

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- للمجلة الحق في تعديل البحث أو طلب تعديله أو رفضه .
- يخضع البحث في النشر لأولويات المجلة وسياساتها .
- البحوث المنشورة تعبر عن وجهة نظر أصحابها ، ولا تعبر عن وجهة نظر المجلة .

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On Coefficient Bounds for Certain Classes of Analytic Functions

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Abstract

In the present paper we obtain Fekete-Szegő inequalities and sharp bounds for certain classes of analytic and p -valent functions in the open unit disk involving certain fractional derivative operator.

Keywords: p -valent function , subordination , starlike function , convex function, fractional derivative operator, Fekete-Szegő inequality.

1. Introduction and Preliminaries

Let $A(p)$ denote the class of functions defined by

$$f(z) = z^p + \sum_{n=1}^{\infty} a_{p+n} z^{p+n}, \quad (p \in \mathbb{N}) \quad (1.1)$$

which are analytic and p -valent in the open unit disk $\mathcal{U} = \{z: |z| < 1\}$.

Let $f(z)$ and $g(z)$ be functions analytic in \mathcal{U} , we say that the function $f(z)$ is subordinate to $g(z)$, if there exists a Schwarz function $w(z)$, analytic in \mathcal{U} , with $w(0) = 0$ and $|w(z)| < 1$ ($z \in \mathcal{U}$), and such that $f(z) = g(w(z))$ for all $z \in \mathcal{U}$.

This subordination is denoted by $f < g$ or $f(z) < g(z)$. It is well known that, if the function $g(z)$ is univalent in \mathcal{U} , then $f(z) < g(z)$ if and only if $f(0) = g(0)$ and $f(\mathcal{U}) \subset g(\mathcal{U})$.

Let $\phi(z)$ be an analytic function with $\phi(0) = 1$, $\phi'(0) > 0$ and $\text{Re}(\phi(z)) > 0$ ($z \in \mathcal{U}$), which maps the open unit disk \mathcal{U} onto a region starlike with respect to 1 and is symmetric with respect to the real axis. Ali et al. [1] defined and studied the class $S_{b,p}^*(\phi)$ to be the class of functions $f(z) \in A(p)$ for which

$$1 + \frac{1}{b} \left\{ \frac{1}{p} \frac{zf'(z)}{f(z)} - 1 \right\} < \phi(z), \quad (z \in \mathcal{U}, b \in \mathbb{C} \setminus \{0\})$$

and the class $C_{b,p}(\phi)$ of all functions for which



$$1 - \frac{1}{b} + \frac{1}{bp} \left\{ 1 + \frac{zf''(z)}{f'(z)} \right\} < \phi(z), \quad (z \in \mathcal{U}, b \in \mathbb{C} \setminus \{0\})$$

Note that $S_{1,1}^*(\phi) = S^*(\phi)$ and $C_{1,1}(\phi) = C(\phi)$, The classes were introduced and studied by Ma and Minda [4]. The familiar class $S^*(\alpha)$ of starlike functions of order α and the class $C(\alpha)$ of convex functions of order α , $0 \leq \alpha < 1$ are the special cases of $S_{1,1}^*(\phi)$ and $C_{1,1}(\phi)$, respectively, when

$$\phi(z) = \frac{1 + (1 - 2\alpha)z}{1 - z}$$

Let ${}_2F_1(a, b; c; z)$ be the Gauss hypergeometric function defined for $z \in \mathcal{U}$ by, (see Srivastava and Karlsson [9]).

$${}_2F_1(a, b; c; z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n n!} z^n$$

where $(\lambda)_n$ is the Pochhammer symbol defined, in terms of the Gamma function, by

$$(\lambda)_n = \frac{\Gamma(\lambda + n)}{\Gamma(\lambda)} = \begin{cases} 1, & n = 0, \\ \lambda(\lambda + 1)(\lambda + 2) \dots (\lambda + n - 1), & n \in \mathbb{N}. \end{cases}$$

for $\lambda \neq 0, -1, -2, \dots$.

We recall the following definitions of fractional derivative operators which were used by Owa [6], (see also [8]) as follows:

Definition 1.1. The fractional derivative operator of order λ is defined by

$$D_z^\lambda f(z) = \frac{1}{\Gamma(1 - \lambda)} \frac{d}{dz} \int_0^z \frac{f(\xi)}{(z - \xi)^\lambda} d\xi, \quad 0 \leq \lambda < 1 \quad (1.2)$$

where $f(z)$ is analytic function in a simply-connected region of the z -plane containing the origin, and the multiplicity of $(z - \xi)^{-\lambda}$ is removed by requiring $\log(z - \xi)$ to be real when $z - \xi > 0$.

Definition 1.2. Let $0 \leq \lambda < 1$, and $\mu, \eta \in \mathbb{R}$. Then, in terms of the familiar Gauss's hypergeometric function ${}_2F_1$, the generalized fractional derivative operator $J_{0,z}^{\lambda, \mu, \eta}$ is

$$J_{0,z}^{\lambda, \mu, \eta} f(z) = \frac{d}{dz} \left(\frac{z^{\lambda - \mu}}{\Gamma(1 - \lambda)} \int_0^z (z - \xi)^{-\lambda} f(\xi) {}_2F_1 \left(\mu - \lambda, 1 - \eta; 1 - \lambda; 1 - \frac{\xi}{z} \right) d\xi \right) \quad (1.3)$$



where $f(z)$ is analytic function in a simply- connected region of the z -plane containing the origin, with the order $f(z) = O(|z|^\varepsilon)$, $z \rightarrow 0$, where $\varepsilon > \max\{0, \mu - \eta\} - 1$ and the multiplicity of $(z - \xi)^{-\lambda}$ is removed requiring $\log(z - \xi)$ to be real when $z - \xi > 0$.

Definition 1.3 Under the hypotheses of Definition 1.2, the fractional derivative operator $J_{0,z}^{\lambda+m, \mu+m, \eta+m}$ of a function $f(z)$ defined by

$$J_{0,z}^{\lambda+m, \mu+m, \eta+m} f(z) = \frac{d^m}{dz^m} J_{0,z}^{\lambda, \mu, \eta} f(z) \quad (1.4)$$

Notice that

$$J_{0,z}^{\lambda, \lambda, \eta} f(z) = D_z^\lambda f(z), \quad 0 \leq \lambda < 1 \quad (1.5)$$

Amsheri and Zharkova [2], (see also [3], [10]) defined the fractional derivative operator $M_{0,z}^{\lambda, \mu, \eta, p} f(z)$ by

$$\begin{aligned} M_{0,z}^{\lambda, \mu, \eta, p} f(z) &= \frac{\Gamma(p+1-\mu)\Gamma(p+1-\lambda+\mu)}{\Gamma(p+1)\Gamma(p+1-\mu+\eta)} z^\mu J_{0,z}^{\lambda, \mu, \eta} f(z) \\ &= z^p + \sum_{n=1}^{\infty} \gamma_n(\lambda, \mu, \eta, p) a_{p+n} z^{p+n} \end{aligned} \quad (1.6)$$

for $f(z) \in A(p)$ and $\lambda \geq 0$; $\mu < p+1$; $\eta > \max(\lambda, \mu) - p - 1$; $p \in \mathbb{N}$, where

$$\gamma_n(\lambda, \mu, \eta, p) = \frac{(p+1)_n (p+1-\mu+\eta)_n}{(p+1-\mu)_n (p+1-\lambda+\eta)_n}, \quad (n \in \mathbb{N}) \quad (1.7)$$

Very recently, Zayed et al. [10] defined the operator $N_{0,z}^{m, \lambda, \mu, \eta, \delta, p}(z) : A(p) \rightarrow A(p)$, for $m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$ and $\delta \geq 0$ as follows:

$$\begin{aligned} N_{0,z}^{0, \lambda, \mu, \eta, \delta, p} f(z) &= M_{0,z}^{\lambda, \mu, \eta, p} f(z) \\ N_{0,z}^{1, \lambda, \mu, \eta, \delta, p} f(z) &= N_{0,z}^{\lambda, \mu, \eta, \delta, p} f(z) \\ &= (1-\delta) M_{0,z}^{\lambda, \mu, \eta, p} f(z) + \delta \frac{z}{p} \left(M_{0,z}^{\lambda, \mu, \eta, p} f(z) \right)' \\ &= z^p + \sum_{n=1}^{\infty} \left(\frac{p+\delta n}{p} \right) \gamma_n(\lambda, \mu, \eta, p) a_{p+n} z^{p+n} \end{aligned}$$

and (in general),

$$N_{0,z}^{m, \lambda, \mu, \eta, \delta, p} f(z) = N_{0,z}^{\lambda, \mu, \eta, \delta, p} (N_{0,z}^{m-1, \lambda, \mu, \eta, \delta, p} f(z))$$



$$= z^p + \sum_{n=1}^{\infty} \left(\frac{p + \delta n}{p} \right)^m \gamma_n(\lambda, \mu, \eta, p) a_{p+n} z^{p+n} \quad (1.8)$$

We let $\gamma_n(\lambda, \mu, \eta, p) \equiv \gamma_n$.

Motivated essentially by aforementioned works, we introduce a more general class of p -valent analytic functions of complex order $S_{b,p,\lambda,\mu,\eta,\delta}^m(\phi)$ which we define in the following.

Definition 1.4 Let $\phi(z)$ be an univalent starlike function with respect to 1 which maps the open unit disk \mathcal{U} onto a region in the right half-plane and symmetric with respect to the real axis, $\phi(0) = 1$ and $\phi'(z) > 0$. A function $f(z) \in A(p)$ is in the class $S_{b,p,\lambda,\mu,\eta,\delta}^m(\phi)$ if

$$1 + \frac{1}{b} \left(\frac{N_{0,z}^{m,\lambda+1,\mu+1,\eta+1,\delta,p} f(z)}{N_{0,z}^{m,\lambda,\mu,\eta,\delta,p} f(z)} - 1 \right) < \phi(z), \quad (z \in \mathcal{U}, b \in \mathbb{C} \setminus \{0\}) \quad (1.9)$$

Also, we let $S_{1,p,\lambda,\mu,\eta,\delta}^m(\phi) = S_{p,\lambda,\mu,\eta,\delta}^m(\phi)$.

The above class $S_{b,p,\lambda,\mu,\eta,\delta}^m(\phi)$ is of special interest and it contains many well-known classes of analytic functions. In particular; for $m = \lambda = \mu = 0$, we have

$$S_{b,p,0,0,\eta,\delta}^0(\phi) = S_{b,p}^*(\phi)$$

where $S_{b,p}^*(\phi)$ is precisely the class which was studied by Ali et al. [1].

Furthermore, by specializing the parameters $b, p, \lambda, \mu, \delta$ and m we obtain the following subclasses which were studied by various other authors:

- 1- For $b = 1, p = 1$ and $\lambda = \mu = \delta = 0$, we get the class $S_{1,1,0,0,\eta,0}^m(\phi) = S^*(\phi)$ which studied by Ma and Minda [4].
- 2- For $p = 1$ and $\lambda = \mu = \delta = 0$, we get the class $S_{b,1,0,0,\eta,0}^m(\phi) = S_b^*(\phi)$ which studied by Ravichandran et al. [7].
- 3- For $b = 1$ and $\lambda = \mu = \delta = 0$, we get the class $S_{1,p,0,0,\eta,0}^m(\phi) = S_p^*(\phi)$ which studied by Ali et al. [1].
- 4- For $m = 0$, we get the class $S_{b,p,\lambda,\mu,\eta,\delta}^0(\phi) = S_{b,\lambda,\mu,\eta,\delta,p}^*(\phi)$ which studied by Amsheri and Zharkova [2].

The motivation of this paper is to give a generalization of the Fekete-Szegő inequalities obtained by Ali et al [1], Ma and Minda [4], Ravichandran et al. [7] and also by Amsheri and Zharkova [2].

Let Ω be the class of analytic functions of the form



$$w(z) = w_1z + w_2z^2 + \dots$$

in the open unit disk \mathcal{U} satisfying $|w(z)| < 1$.

In order to prove our results, we need the following results which shall be used in the sequel.

Lemma 1.5 [1]. If $w \in \Omega$, then

$$|w_2 - tw_1^2| \leq \begin{cases} -t & \text{if } t \leq -1, \\ 1 & \text{if } -1 \leq t \leq 1, \\ t & \text{if } t \geq 1. \end{cases}$$

when $t < -1$ or $t > 1$, the equality holds if and only if $w(z) = z$ or one of its rotations. If $-1 < t < 1$, then equality holds if and only if $w(z) = z^2$ or one of its rotations. Equality holds for $t = -1$ if and only if

$$w(z) = z \frac{\lambda + z}{1 + \lambda z}, \quad (0 \leq \lambda \leq 1)$$

or one of its rotations, while for $t = 1$, the equality holds if and only if

$$w(z) = -z \frac{\lambda + z}{1 + \lambda z}, \quad (0 \leq \lambda \leq 1)$$

or one of its rotations .

Although the above upper bound is sharp, it can be improved as follows when $-1 < t < 1$:

$$|w_2 - tw_1^2| + (t + 1)|w_1|^2 \leq 1, \quad (-1 < t \leq 0)$$

and

$$|w_2 - tw_1^2| + (1 - t)|w_1|^2 \leq 1, \quad (0 < t < 1).$$

Lemma 1.6 [5]. If $w \in \Omega$, then for any complex number t ,

$$|w_2 - tw_1^2| \leq \max(1, |t|).$$

The result is sharp for the functions $w(z) = z$ or $w(z) = z^2$.

2. Coefficient Bounds

By making use of Lemmas 1.5-1.6, we prove the following:



Theorem 2.1. Let $0 \leq \theta \leq 1$; $\lambda \geq 0$; $\mu < p + 1$; $\eta > \max(\lambda, \mu) - p - 1$, $m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$; $\delta \geq 0$ and $p \in \mathbb{N}$. Further, let $\phi(z) = 1 + B_1z + B_2z^2 + \dots$, where B_n 's are real with $B_1 > 0$, $B_2 \geq 0$, and

$$\sigma_1 = \frac{[(B_2 - B_1) + (p - \mu)B_1^2]\gamma_1^2 \left(\frac{p + \delta}{p}\right)^{2m}}{2\gamma_2 B_1^2 (p - \mu) \left(\frac{p + 2\delta}{p}\right)^m}, \quad (2.1)$$

$$\sigma_2 = \frac{[(B_2 + B_1) + (p - \mu)B_1^2]\gamma_1^2 \left(\frac{p + \delta}{p}\right)^{2m}}{2\gamma_2 B_1^2 (p - \mu) \left(\frac{p + 2\delta}{p}\right)^m}, \quad (2.2)$$

$$\sigma_3 = \frac{\left(\frac{p + \delta}{p}\right)^{2m} [B_2 + (p - \mu)B_1^2]}{2\gamma_2 B_1^2 (p - \mu) \left(\frac{p + 2\delta}{p}\right)^m}. \quad (2.3)$$

If $f(z)$ given by (1.1) belongs to $S_{1,p,\lambda,\mu,\eta,\delta}^m(\phi)$, then

$$\begin{aligned} & |a_{p+2} - \theta a_{p+1}^2| \\ & \leq \begin{cases} \frac{(p - \mu)}{2\gamma_2 \left(\frac{p + 2\delta}{p}\right)^m} \left(B_2 - \frac{(p - \mu) \left(2\gamma_2 \theta \left(\frac{p + 2\delta}{p}\right)^m - \gamma_1^2 \left(\frac{p + \delta}{p}\right)^{2m} \right)}{\gamma_1^2 \left(\frac{p + \delta}{p}\right)^{2m}} B_1^2 \right), & \theta \leq \sigma_1, \\ \frac{(p - \mu)B_1}{2\gamma_2 \left(\frac{p + 2\delta}{p}\right)^m}, & \sigma_1 \leq \theta \leq \sigma_2, \\ -\frac{(p - \mu)}{2\gamma_2 \left(\frac{p + 2\delta}{p}\right)^m} \left(B_2 - \frac{(p - \mu) \left(2\gamma_2 \theta \left(\frac{p + 2\delta}{p}\right)^m - \gamma_1^2 \left(\frac{p + \delta}{p}\right)^{2m} \right)}{\gamma_1^2 \left(\frac{p + \delta}{p}\right)^{2m}} B_1^2 \right), & \theta \geq \sigma_2. \end{cases} \end{aligned} \quad (2.4)$$

Further, if $\sigma_1 \leq \theta \leq \sigma_3$, then



$$|a_{p+2} - \theta a_{p+1}^2| + \frac{\gamma_1^2 \left(\frac{p+\delta}{p}\right)^{2m}}{2\gamma_2 B_1 (p-\mu) \left(\frac{p+2\delta}{p}\right)^m} \cdot \left(1 - \frac{B_2}{B_1} + \frac{(p-\mu) \left(2\gamma_2 \theta \left(\frac{p+2\delta}{p}\right)^m - \gamma_1^2 \left(\frac{p+\delta}{p}\right)^{2m}\right)}{\gamma_1^2 \left(\frac{p+\delta}{p}\right)^{2m}} B_1\right) |a_{p+1}|^2 \leq \frac{(p-\mu)B_1}{2\gamma_2 \left(\frac{p+2\delta}{p}\right)^m} \quad (2.5)$$

If $\sigma_3 \leq \theta \leq \sigma_2$, then

$$|a_{p+2} - \theta a_{p+1}^2| + \frac{\gamma_1^2 \left(\frac{p+\delta}{p}\right)^{2m}}{2\gamma_2 B_1 (p-\mu) \left(\frac{p+2\delta}{p}\right)^m} \cdot \left(1 + \frac{B_2}{B_1} - \frac{(p-\mu) \left(2\gamma_2 \theta \left(\frac{p+2\delta}{p}\right)^m - \gamma_1^2 \left(\frac{p+\delta}{p}\right)^{2m}\right)}{\gamma_1^2 \left(\frac{p+\delta}{p}\right)^{2m}} B_1\right) |a_{p+1}|^2 \leq \frac{(p-\mu)B_1}{2\gamma_2 \left(\frac{p+2\delta}{p}\right)^m} \quad (2.6)$$

For any complex number θ ,

$$|a_{p+2} - \theta a_{p+1}^2| \leq \frac{(p-\mu)B_1}{2\gamma_2 \left(\frac{p+2\delta}{p}\right)^m} \max \left\{ 1, \left| \frac{(p-\mu) \left(2\gamma_2 \theta \left(\frac{p+2\delta}{p}\right)^m - \gamma_1^2 \left(\frac{p+\delta}{p}\right)^{2m}\right)}{\gamma_1^2 \left(\frac{p+\delta}{p}\right)^{2m}} B_1 - \frac{B_2}{B_1} \right| \right\} \quad (2.7)$$

The results are sharp.

Proof. If $f(z) \in S_{1,p,\lambda,\mu,\eta,\delta}^m(\phi)$, then there is a Schwarz function

$$w(z) = w_1 z + w_2 z^2 + \dots \in \Omega$$

such that

$$\frac{N_{0,z}^{m,\lambda+1,\mu+1,\eta+1,\delta,p} f(z)}{N_{0,z}^{m,\lambda,\mu,\eta,\delta,p} f(z)} = \phi(w(z)) \quad (2.8)$$



since

$$\frac{N_{0,z}^{m,\lambda+1,\mu+1,\eta+1,\delta,p} f(z)}{N_{0,z}^{m,\lambda,\mu,\eta,\delta,p} f(z)} = 1 + \frac{\gamma_1}{(p-\mu)} \left(\frac{p+\delta}{p}\right)^m a_{p+1} z + \frac{1}{(p-\mu)} \left[2\gamma_2 \left(\frac{p+2\delta}{p}\right)^m a_{p+2} - \gamma_1^2 \left(\frac{p+\delta}{p}\right)^{2m} a_{p+1}^2 \right] z^2 + \dots$$

We have from (2.8),

$$a_{p+1} = \frac{(p-\mu)w_1}{\gamma_1 \left(\frac{p+\delta}{p}\right)^m} B_1, \quad (2.10)$$

and

$$a_{p+2} = \frac{(p-\mu)}{2\gamma_2 \left(\frac{p+2\delta}{p}\right)^m} \{B_1 w_2 + (B_2 + (p-\mu)B_1^2)w_1^2\} \quad (2.11)$$

Therefore, we have

$$a_{p+2} - \theta a_{p+1}^2 = \frac{(p-\mu)B_1}{2\gamma_2 \left(\frac{p+2\delta}{p}\right)^m} \{w_2 - v w_1^2\} \quad (2.12)$$

where

$$v := \frac{(p-\mu)B_1 \left(2\gamma_2 \theta \left(\frac{p+2\delta}{p}\right)^m - \gamma_1^2 \left(\frac{p+\delta}{p}\right)^{2m} \right)}{\gamma_1^2 \left(\frac{p+\delta}{p}\right)^{2m}} - \frac{B_2}{B_1} \quad (2.13)$$

The results (2.4)-(2.7) are established by an application of Lemma 1.5 and inequality (2.7) by Lemma 1.6.

To show that the bounds in (2.4)-(2.7) are sharp, we define the functions $K_{\phi n}$ ($n = 2, 3, \dots$) by

$$\frac{N_{0,z}^{m,\lambda+1,\mu+1,\eta+1,\delta,p} K_{\phi n}(z)}{N_{0,z}^{m,\lambda,\mu,\eta,\delta,p} K_{\phi n}(z)} = \phi(z^{n-1}), \quad K_{\phi n}(0) = (K_{\phi n})'(0) - 1 = 0$$

and the functions F_r, G_r ($0 \leq r \leq 1$) defined by



$$\frac{N_{0,z}^{m,\lambda+1,\mu+1,\eta+1,\delta,p} F_r(z)}{N_{0,z}^{m,\lambda,\mu,\eta,\delta,p} F_r(z)} = \phi \left(\frac{z(z+r)}{1+rz} \right), \quad F_r(0) = F_r'(0) - 1 = 0$$

and

$$\frac{N_{0,z}^{m,\lambda+1,\mu+1,\eta+1,\delta,p} G_r(z)}{N_{0,z}^{m,\lambda,\mu,\eta,\delta,p} G_r(z)} = \phi \left(-\frac{z(z+r)}{1+rz} \right), \quad G_r(0) = G_r'(0) - 1 = 0$$

respectively, it is clear that the functions K_{ϕ_n} , F_r and G_r belong to the class $S_{1,p,\lambda,\mu,\eta,\delta}^m(\phi)$. If $\theta < \sigma_1$ or $\theta > \sigma_2$, then the equality holds if and only if f is K_{ϕ_2} or one of its rotations. If $\sigma_1 < \theta < \sigma_2$, the equality holds if and only if f is K_{ϕ_3} or one of its rotations. If $\theta = \sigma_1$, then the equality holds if and only if f is F_r or one of its rotations. If $\theta = \sigma_2$, then the equality holds if and only if f is G_r or one of its rotations.

Theorem 2.2. Let $\lambda \geq 0$; $\mu < p + 1$; $\eta > \max(\lambda, \mu) - p - 1$, $b \in \mathbb{C} \setminus \{0\}$; $m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$; $\delta \geq 0$ and $p \in \mathbb{N}$. Further, let $\phi(z) = 1 + B_1 z + B_2 z^2 + \dots$, where B_n 's are real with $B_1 > 0$, $B_2 \geq 0$. If $f(z)$ given by (1.1) belongs to $S_{b,p,\lambda,\mu,\eta,\delta}^m(\phi)$, then for any complex number θ , we have

$$|a_{p+2} - \theta a_{p+1}^2| \leq \frac{(p-\mu)|b|B_1}{2\gamma_2 \left(\frac{p+2\delta}{p}\right)^m} \max \left\{ 1, \left| \frac{(p-\mu)b \left(2\gamma_2 \theta \left(\frac{p+2\delta}{p}\right)^m - \gamma_1^2 \left(\frac{p+\delta}{p}\right)^{2m} \right)}{\gamma_1^2 \left(\frac{p+\delta}{p}\right)^{2m}} B_1 - \frac{B_2}{B_1} \right| \right\}. \quad (2.14)$$

The result is sharp.

Proof. The proof is similar to the proof of Theorem 2.1.

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