مجـلة الـتربـوي
Journal of Educational
ISSN: 2011- 421X
Arcif Q3

معامل التأثير العربي 1.5
العدد 20

مجلة التربوي

مجلة علمية محكمة تصدر عن كلية التربية

جامعة المرقب

العدد العشرون
يناير 2022م

هيئـــة تحريـر
مجلة التربوي

- المجلة ترحب بما يرد عليها من أبحاث وعلى استعداد لنشرها بعد التحكيم .
- المجلة تحترم كل الاحترام آراء المحكمين وتعمل بمقتضاها .
- كافة الآراء والأفكار المنشورة تعبر عن آراء أصحابها ولا تتحمل المجلة تبعاتها .
- يتحمل الباحث مسؤولية الأمانة العلمية وهو المسؤول عما ينشر له .
- البحوث المقدمة للنشر لا ترد لأصحابها نشرت أو لم تنشر .

مجـلة الـتربـوي
Journal of Educational
ISSN: 2011- 421X
Arcif Q3

معامل التأثير العربي 1.5
العدد 20

ضوابط النشر:

يشترط في البحوث العلمية المقدمة للنشر أن يراعى فيها ما يأتي :

- أصول البحث العلمي وقواعده .
- ألا تكون المادة العلمية قد سبق نشرها أو كانت جزءا من رسالة علمية .
- يرفق بالبحث تزكية لغوية وفق أنموذج معد .
- تعدل البحوث المقبولة وتصحح وفق ما يراه المحكمون .
- التزام الباحث بالضوابط التي وضعتها المجلة من عدد الصفحات ، ونوع الخط ورقمه ، والفترات الزمنية الممنوحة للتعديل ، وما يستجد من ضوابط تضعها المجلة مستقبلا .

تنبيهات :

- للمجلة الحق في تعديل البحث أو طلب تعديله أو رفضه .
- يخضع البحث في النشر لأولويات المجلة وسياستها .
- البحوث المنشورة تعبر عن وجهة نظر أصحابها ، ولا تعبر عن وجهة نظر المجلة .

Information for authors

1- Authors of the articles being accepted are required to respect the regulations and the rules of the scientific research.
2- The research articles or manuscripts should be original and have not been published previously. Materials that are currently being considered by another journal or is a part of scientific dissertation are requested not to be submitted.
3- The research articles should be approved by a linguistic reviewer.
4- All research articles in the journal undergo rigorous peer review based on initial editor screening.
5- All authors are requested to follow the regulations of publication in the template paper prepared by the editorial board of the journal.

Attention
1- The editor reserves the right to make any necessary changes in the papers, or request the author to do so, or reject the paper submitted.
2- The research articles undergo to the policy of the editorial board regarding the priority of publication.
3- The published articles represent only the authors' viewpoints.

مجـلة الـتربـوي
Journal of Educational
ISSN: 2011- 421X
Arcif Q3

معامل التأثير العربي 1.5
العدد 20

Conjugate Newton's Method for a Polynomial of degree m+1

Omar Ismail Elhasadi[1], Mohammed Saleh Alsayd[2] and Elhadi A. A. Maree[3]
Department of Mathematics, School of basic Sciences, The Libyan Academy for Graduate Studies, Tripoli[1,2]
Department of Mathematics, Faculty of Science, Elmergib University[3]
omar.ismail@academy.edu.ly[1]

0.1 Abstract

The main problem of this paper is to conjugate the cubic equation to quadratic equation by using Möbius transformation, so the work to finding the roots of polynomials with Complex variable and double roots using Newton’s method will be easier. Also we make some correction to the linear fractional transformation (or Mobius transformation) which mentioned in [12].

0.2 Introduction

In this paper is devoted to investigate the using of the complex dynamic Newton's Method for a double Root. We discuss different cases including: simple root and multiple roots. To approximating the real or complex roots of a polynomial function $g(z)$, Newton's Method which consists of iterating the function $N(z)$ is needed where;

$$N(z) = z - \frac{g(z)}{g'(z)}. \qquad (0.2.1)$$

If we chose the , initial value for z_0 to be sufficiently close to a root of g, the sequence of iterations , $z_{k+1} = N(z_k)$ will converge to the root. The type of convergence depends on the complexity of the root, if the root is simple, the sequence converges quadratically, but if the root is a multiple, the convergence can be only linear. Newton's method can be adjusted to have a better convergence to multiple roots. If the function $g(z)$ has a multiple root of order exactly m, the applying Newton's method to $\sqrt[m]{g(z)}$ leads to

$$N_m(z) = z - \frac{\sqrt[m]{g(z)}}{\left(\sqrt[m]{g(z)}\right)'} \,,$$

$$N_m(z) = z - \frac{\left(g(z)\right)^{\frac{1}{m}}}{\frac{1}{m} g(z)^{\frac{1}{m}-1} g'(z)} \,,$$

$$= \frac{[\left(g(z)\right)^{\frac{1}{m}-1} . g'(z)]z - m\left(g(z)\right)^{\frac{1}{m}}}{g(z)^{\frac{1}{m}-1} . g'(z)} \,,$$

we get;

$$N_m(z) = \frac{\left(g(z)\right)^{\frac{1}{m}-1}[g'(z)z - mg(z)]}{\left(g(z)\right)^{\frac{1}{m}-1} . g'(z)} \,,$$

$$N_m(z) = \frac{g'(z)z - mg(z)}{g'(z)} = z - \frac{mg(z)}{g'(z)} \,, \qquad (0.2.2)$$

This method is called relaxed Newton's method or Newton's method for a root of order m. This method converge quadratically to a root of order m.

Newton's method defines a dynamical system on the complex Riemann sphere for each polynomial function $g(z)$. Peitgen and Haeseler provide with an overview of the complex dynamics of Newton's method for a rational function , and discuss the basins of attraction of the roots for the relaxed Newton's method. We introduced how the relaxed Newton's method for a double root is applied to a cubic with a double root, and illustrate that the dynamics is conjugate to that of a well-known Julia set. Applying the relaxed Newton's method for a root of order m. to a polynomial of degree $m + 1$. Came up with a similar result. Also, it is shown that the Julia sets came by Bums, Palmore and Benzinger applying the standard Newton's method to the family of functions $(z - 1)(z + \alpha)^k$ are conjugate to the Julia sets of quadratics.

0.3 Conjugate the Map N by Transformation to obtain a Quadratic Polynomial

Theorem 1.

For the rational map $N_p(z)$ arising from Newton's method applied to a quadratic polynomial $p(z) = (z - a)(z - b), a \neq b,$ $N_p(z)$ is conjugate to z^2 by the Möbius transformation.

Proof:

By substitution in Eq. (0.2.1) we obtaining

$$N(z) = z - \frac{(z-a)(z-b)}{(z-a)+(z-b)},$$

$$= \frac{z[(z-a)+(z-b)] - (z-a)(z-b)}{(z-a)+(z-b)},$$

$$= \frac{z^2 - az + z^2 - bz - z^2 + bz + az - ab}{2z - a - b},$$

$$N(z) = \frac{z^2 - ab}{2z - a - b}, \qquad (0.3.1)$$

by derivative Eq. (0.3.1) we get

$$N'(z) = \frac{(2z-a-b).2z - (z^2 - ab).2}{(2z-a-b)^2},$$

$$= \frac{4z^2 - 2az - 2bz - 2z^2 + 2ab}{(2z-a-b)^2},$$

$$= \frac{2z^2 - 2az - 2bz + 2ab}{(2z-a-b)^2},$$

$$N'(z) = \frac{2(z^2 - az - bz + ab)}{(2z-a-b)^2},$$

The fixed point of N are $z = a$ and $z = b$, since $N'(a) = N'(b) = 0$ then a and b are a super attractive fixed points.

The critical point of N are $z = a$ and $z = b$.

The critical points of the quadratic $p(z) = z^2 + c$ on the Riemann sphere are $z = 0$ and $z = \infty$ is also a super attractive fixed point.

the linear fractional transformation (or Möbius transformation). Using the transformation

$$h(z) = \frac{z-p}{z-q}\,, \qquad (0.3.2)$$

Where p is turns to zero and q is turns to ∞.

Now we have $h(z) = \frac{z-a}{z-b}$,

to find $h^{-1}(z)$, assume that

$$h^{-1}(z) = w \;\rightarrow z = h(w)\,,$$

$$z = \frac{w-a}{w-b}\,,$$

$$\Longrightarrow \quad (w-b)z = (w-a),$$

$$\Longrightarrow \quad wz - bz = w - a,$$

$$wz - w = bz - a,$$

$$w(z-1) = bz - a,$$

$$w = \frac{(bz-a)}{z-1} \Longrightarrow h^{-1}(z) = \frac{bz-a}{z-1}\,,$$

Conjugate the map N, by transformation h, to obtain the quadratic;

Let ; $p(z) = h \circ N \circ h^{-1}(z)\,,$

$$N \circ h^{-1}(z) = \frac{z^2 - ab}{2z - a - b} \circ \frac{bz-a}{z-1} = \frac{\left(\frac{bz-a}{z-1}\right)^2 - ab}{2\left(\frac{bz-a}{z-1}\right) - a - b},$$

$$= \frac{[(bz-a)^2 - ab(z-1)^2]}{[2(bz-a)(z-1) - (a+b)(z-1)^2]}\,,$$

$$= \frac{[b^2z^2 - 2abz + a^2 - abz^2 + 2abz - ab]}{[2bz^2 - 2bz - 2az + 2a - az^2 + 2az - a - bz^2 + 2bz - b]},$$

$$= \frac{b^2z^2 + a^2 - abz^2 - ab}{bz^2 + a - az^2 - b},$$

$$= \frac{(bz^2 - a)[b - a]}{(z^2 - 1)[b - a]} = \frac{bz^2 - a}{z^2 - 1},$$

therefore

$$h \circ N \circ h^{-1}(z) = \frac{z - a}{z - 1} \circ \frac{bz^2 - a}{z^2 - 1},$$

$$\Rightarrow \quad \frac{\frac{bz^2 - a}{z^2 - 1} - a}{\frac{bz^2 - a}{z^2 - 1} - b} = \frac{bz^2 - a - a(z^2 - 1)}{bz^2 - a - b(z^2 - 1)},$$

$$= \frac{bz^2 - a - az^2 + a}{bz^2 - a - bz^2 + b} = \frac{z^2(b - a)}{b - a} = z^2.$$

Theorem 2.

The relaxed Newton's method N_2 for a double root applied to any cubic equation with a double root is conjugate by a linear fractional transformation on the Riemann sphere to the iterations of the quadratic

$$p(z) = \frac{3z^2 - 1}{2}$$

Proof:

Assume that cubic be $g(z) = (z - a)^2(z - b)$, where a and b are distinct complex numbers. By applying the Newton's method for a double root to a this cubic, we obtain the following;

$$N_2(z) = z - \frac{2g(z)}{g'(z)},$$

$$N_2(z) = z - \frac{2[(z - a)^2(z - b)]}{[(z - a)^2(z - b)]'},$$

$$= z - \frac{2[(z-a)(z-a)(z-b)]}{[(z-a)^2.1 + (z-b).2(z-a).1]},$$

$$= z - \frac{(z-a)[3(z-a)(z-b)]}{(z-a)[(z-a) + 2(z-b)]},$$

$$= z - \frac{2[(z-a)(z-b)]}{[(z-a) + 2(z-b)]},$$

$$= \frac{z^2 - az + 2z^2 - 2bz - 2z^2 + 2bz + 2az - 2ab}{z - a + 2z - 2b},$$

$$N_2(z) = \frac{z^2 + az - 2ab}{3z - a - 2b}, \qquad (0.3.3)$$

by derivative Eq. (0.3.3) we obtaining

$$N_2'(z) = \frac{(3z-a-2b)[2z+a] - [(z^2+az-2ab).3]}{(3z-a-2b)^2},$$

$$= \frac{6z^2 + 3az - 2az - a^2 - 4bz - 2ab - 3z^2 - 3az + 6ab}{(3z-a-2b)^2},$$

$$= \frac{3z^2 - 2az - a^2 - 4bz + 4ab}{(3z-a-2b)^2},$$

$$N_2'(z) = \frac{(z-a)[3z+a-4b]}{(3z-a-2b)^2}. \qquad (0.3.4)$$

It is clear that N_2 has fixed points of $z = a$ and $z = b$. when $z = a$ the derivative $N_2'(a) = 0$,So, a is a super attractive fixed point, and when

$z = b$ the $N_2'(b) = -1$, then ,b is a neutral fixed point.

The critical points of $N_2'(z) = 0$,

To find critical points, we put $N_2'(z) = 0$ in Eq. (0.3.4)

$$\frac{(z-a)(3z+a-4b)}{(3z-a-2b)^2} = 0 \rightarrow (z-a)(3z+a-4b) = 0,$$

$$z - a = 0, \Longrightarrow z = a$$

$$or\ \ 3z + a - 4b = 0,$$

$$3z = 4b - a, \qquad \Longrightarrow z = \frac{4b - a}{3},$$

The critical points of the quadratic $p(z) = z^2 + c$ on the Riemann sphere;

$$p'(z) = 2z\,,$$

To find critical point we put $p'(z) = 0$;

$$2z = 0 \ \Longrightarrow z = 0, \qquad or\ \ z = \infty,$$

Since $z = \infty$ is a super attractive fixed point too.

Mobius transformation on the linear fractional transformation, by using transformation in Eq. (0.3.2)

where p is to be transform to zero, and q is to be transform to ∞,

$$h(z) = \frac{z - \left(\frac{4b - a}{3}\right)}{z - a},$$

$$= \frac{3z - 4b + a}{3z - 3a},$$

$$h(z) = \frac{3z - 4b + a}{3(z - a)}.$$

to find $h^{-1}(z)$, we suppose that;

$$h^{-1}(z) = w \ \rightarrow z = h(w),$$

$$z = \frac{3w + a - 4b}{3(w - a)},$$

$$3z(w - a) = 3w + a - 4b,$$

$$3zw - 3za = 3w + a - 4b,$$

$$3zw - 3w = 3az + a - 4b,$$

$$w(3z - 3) = 3az + a - 4b,$$

$$h^{-1}(z) = w = \frac{3az + a - 4b}{3z - 3},$$

Sends a to ∞ and $(4b - a)/3$ to 0, and map the critical points of N_2 to the critical points of the quadratic p.use the transformation h to Conjugate the map N_2, by the transformation h, to obtain the quadratic, using MATLAB the result could be obtained as:

$$p(z) = h \circ N_2 \circ h^{-1}(z) = \frac{3z^2 - 1}{2}.$$

We chose The constant factor of h was chosen so that the coefficient of z^2 became 3/2.

Therefore , we found that under the map h on the Riemann sphere, the dynamics of the relaxed Newton's method N_2 is conjugate to the dynamics of p.

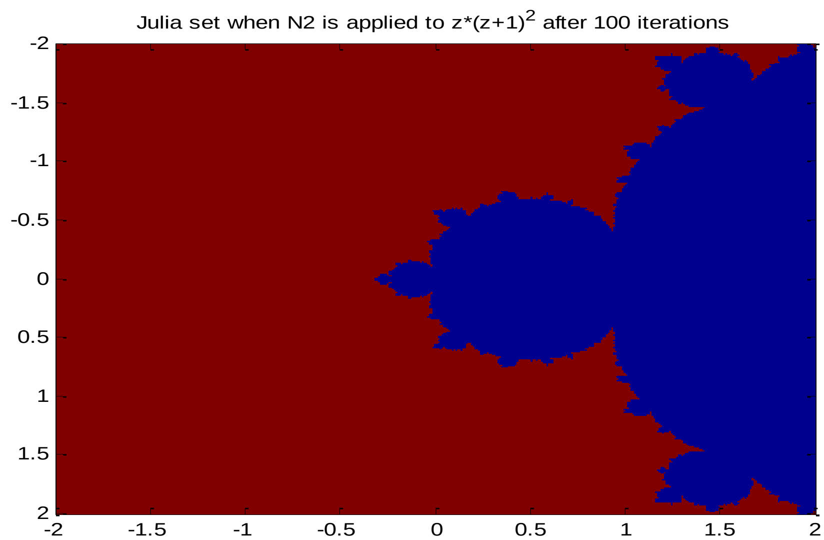

Figure 1: the Brown area is the basin of attraction, after 100 iterations, of the double root -1, when N_2 is applied to $(z+1)^2 z$.

The previous Figure 1 illustrate the basin of attraction for the double root in the complex plane when the relaxed Newton's method N_2 is applied to the cubic $g(z) = z(z+1)^2$. All initial values in the brown region will converge to the double root $a = -1$, while initial values the blue region do not converge to within 0.001 of a root in 100 iterations.

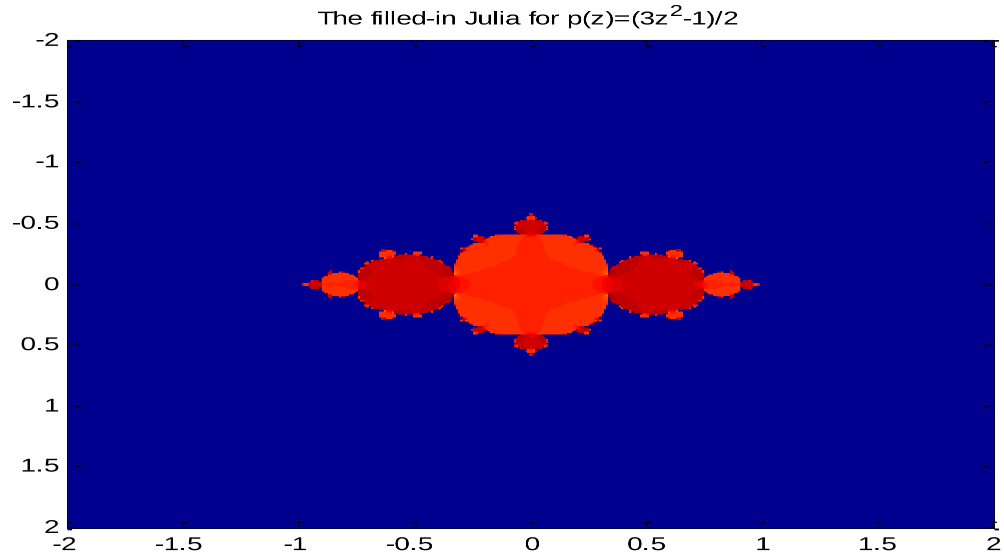

Figure 2: The filled-in Julia set for $p(z) = (3z^2 - 1)/2$

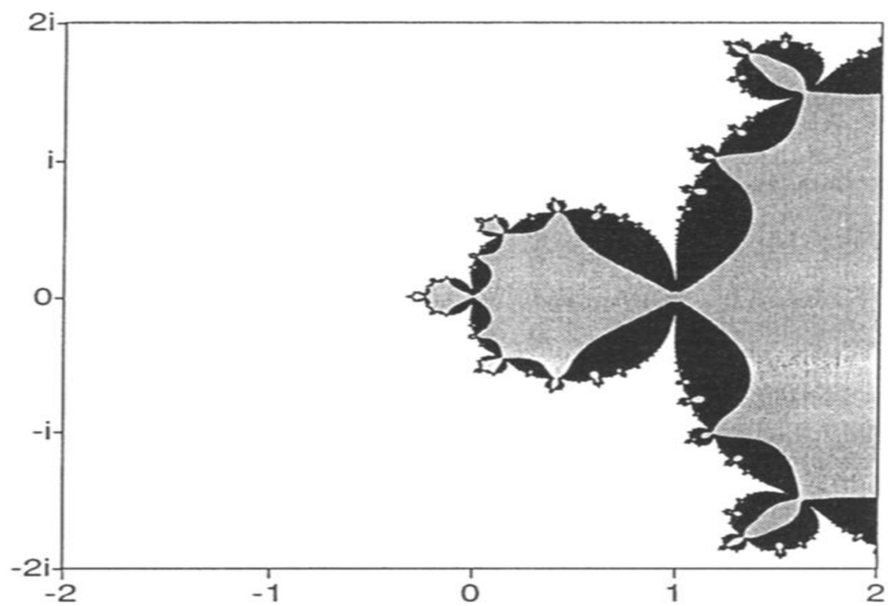

Figure 3: The same as Figure 1, except after 1195 iterations points in the gray area come to within 0.01 of the simple root 0.

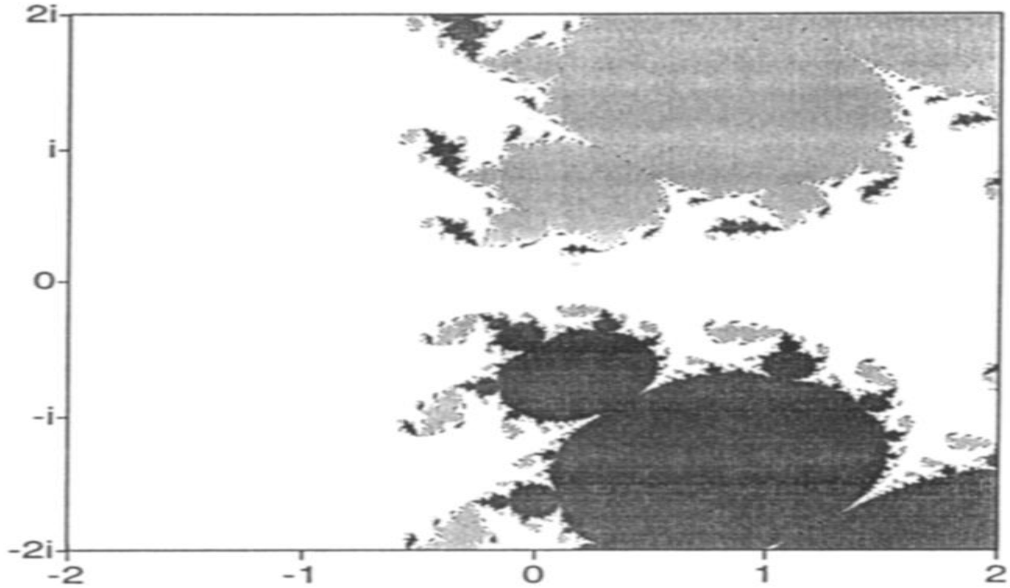

Figure 4: the basins of attraction for N_2 applied to $p(z) = (z+1)^2(z^2+0.25)$. After 1000 iterations the point shaded white, light gray and dark gray, come to within 0.01 of the roots $-1,\ i/2$ and $-i/2$, respectively.

Hence the points inside the blue region in figure 1 do converge to the root $b = 0$, but do so extremely slowly, as shown in figure 3. However, there is no complete neighborhood of b in which points all converge to b, since any such neighborhood intersects the Julia set. Points starting on the Julia set will not converge, but will remain on the boundary under iterations of N_2.

Benzinger, Burns, and Palmore discuss Newton's method for the family of functions;

$$f_\alpha(z) = (z+\alpha)^\alpha(z-1),$$

For $\alpha = 1/2$, this gives the function $(z-1)\sqrt{z+0.5}$ and the basins of attraction for Newton's method, this is as the same our figure 1, since it follows from equation (0.2.2) that the relaxed Newton's method, N_2, for $(z-1)^2(z+0.5)$ is the same as Newton's method, N for $(z-1)\sqrt{z+0.5}$.

0.4 Conjugate polynomial of degree $m+1$ and using Newton's method to find the roots

To investigate more generally what happens to the relaxed Newton's method, apply N_m to the function $g(z)=q(z)(z-b)^k$. Assume that b is not a root of q and assume that q has finite derivative. The root b has order k and;

$$N_m(z)=z-\frac{mg(z)}{g'(z)},$$

$$g(z)=q(z)(z-b)^k,$$

$$N_m(z)=z-\frac{m(q(z)(z-b)^k)}{q(z).k(z-b)^{k-1}+(z-b)^k q'(z)},$$

Then we get;

$$N_m(z)=z-\frac{mq(z)(z-b)^{k-1}.(z-b)}{(z-b)^{k-1}[kq(z)+(z-b)q'(z)]},$$

$$N_m(z)=z-\frac{mq(z).(z-b)}{kq(z)+(z-b)q'(z)},$$

New, we find $N'_m(z)$;

$$N'_m(z)=1-\frac{[kq(z)+(z-b)q'(z)]m[kq(z)+(z-b)q'(z)]}{[kq(z)+(z-b)q'(z)]^2}$$

$$-\frac{m[q(z)(z-b).kq'(z)+(z-b)''q(z).q'(z)]}{[kq(z)+(z-b)q'(z)]^2},$$

$$N'_m(b)=1-\frac{mkq^2(b)}{k^2q(b)} \Longrightarrow N'_m(b)=1-\frac{m}{k}.$$

So that b is a super attractive fixed point only if $k=m$. Otherwise, if $k<\frac{m}{2}$, $|N'_m(b)|>1$ and b is a repulsive fixed point. If $k=\frac{m}{2}$, $|N'_m(b)|=1$ and b is a neutral fixed point, as the cubic above shows. If $k>\frac{m}{2}$, $|N'_m(b)|<1$ and b is an attractive fixed point for N_m, but the convergence is only linear, not quadratic.

In the typical situation where we apply the relaxed Newton's method, N_2 to a function with one double root and no other multiple roots, all the simple roots will be neutral fixed points. For example, figure 4 shown the basins of attraction for a quartic with one double root. Theorem (3) and its proof can be easily generalized to the relaxed Newton's method N_m.

Theorem 3.

The relaxed Newton's method N_m applied to any polynomial of degree $(m+1)$ with a root of order m, is conjugate by a linear fractional transformation on the Riemann sphere, to the iterations of the quadratic;

Proof:

$$\because g(z) = (z-a)^m (z-b),$$

$$N_m(z) = z - \frac{mg(z)}{g'(z)},$$

$$N_m(z) = z - \frac{m[(z-a)(z-b)]}{(z-a)^m + m(z-a)^{m-1}(z-b)},$$

$$= z - \frac{m[(z-a)(z-b)]}{(z-a)^{m-1}[(z-a)+m(z-b)]},$$

$$= z - \frac{m[(z-a)(z-b)]}{(z-a)+m(z-b)},$$

$$= \frac{z^2 - az + amz - abm}{z - a + mz - mb}, \qquad (0.4.1)$$

by derivative Eq. (0.4.1), we get

$$N'_m(z) = \frac{(z-a+mz-bm)[2z-a+am]}{(z-a+mz-mb)^2}$$

$$-\frac{[(z^2-az+amz-abm)(1+m)]}{(z-a+mz-mb)^2},$$

$$= \frac{z^2 - 2az + a^2 - a^2m + mz^2 - 2bmz + 2abm}{(z-a+mz-mb)^2},$$

If we used the long division we find $z = a$ it be exercise the numerator, then we get;

$$N'_m(z) = \frac{(z-a)[(z-a+am-2bm+mz)]}{(z-a+mz-mb)^2},$$

After that, we find the critical points for N'_m

$$\because N'_m(z) = \frac{(z-a)[(z-a+am-2bm+mz)]}{(a-z+bm-mz^2)},$$

We put $N'_m = 0$

$$(z-a)[(z-a+am-2bm+mz)] = 0,$$

$$(z-a) = 0 \Longrightarrow z = a,$$

$$or \quad [(z-a+am-2bm+mz)] = 0,$$

$$z + mz = a - am + 2bm,$$

$$z = \frac{a-am+2bm}{1+m} \quad , \quad m \neq -1,$$

Now, we use the transformation

$$h(z) = \frac{z-p}{z-q},$$

where p is to be transform to zero, and q is to be transform to ∞,

$$h_m(z) = \frac{z - \left(\frac{a-am+2bm}{1+m}\right)}{z-a},$$

$$h_m(z) = \frac{\frac{z+mz-a+am-2bm}{1+m}}{z-a},$$

$$= \frac{z+mz-a+am-2bm}{(1+m)(z-a)},$$

Then, we find h_m^{-1};

$$h_m^{-1}(z) = w \rightarrow z = h_m(w),$$

$$z = \frac{w+mw-a+am-2bm}{(1+m)(w-a)},$$

$$z(1+m)(w-a) = w + mw - a + am - 2bm,$$

$$wz - az + mzw - amz = w + mw - a + am - 2bm,$$

$$wz + mwz - w - mw = az + amz - a + am - 2bm,$$

$$w(z + mz - 1 - m) = az + amz - a + am - 2bm,$$

$$w = \frac{az + amz - a - am - 2bm}{(z + mz - 1 - m)}$$

$$= \frac{az + amz - a - am - 2bm}{(z-1) + m(z-1)},$$

$$w = \frac{az + amz - a - am - 2bm}{(z-1)(1+m)} = h_m^{-1}(z),$$

And by using the MATLAB, the result could be obtained as:

$$p(z) = h \circ N_m \circ h^{-1} = \frac{(mz^2 - m + z^2 + 1)}{2},$$

In particular, when $m = 3$, the Julia set of $2z^2 - 1$ is the line segment on the real axis between -1 and 1. Then;

$$p(z) = (z-a)^3(z-b),$$

$$N_3(z) = z - \frac{3g(z)}{g'(z)},$$

$$N_3(z) = z - \frac{3[(z-a)^3(z-b)]}{[(z-a)^3(z-b)]'},$$

$$= z - \frac{3[(z-a)(z-a)(z-a)(z-b)]}{[(z-a)^3.1 + (z-b).3(z-a)^2.1]},$$

$$= z - \frac{3[(z-a)^2(z-a)(z-b)]}{(z-a)^2[(z-a) + 3(z-b)]},$$

$$= z - \frac{3(z-a)^2[(z-a)(z-b)]}{(z-a)^2[(z-a) + 3(z-b)]},$$

$$= z - \frac{3[(z-a)(z-b)]}{z-a+3z-3b},$$

$$= z - \frac{3[z^2-bz-az+ab]}{4z-a-3b},$$

$$= \frac{4z^2-az-3bz-3z^2+3bz+3az-3ab}{4z-a-3b},$$

$$N_3(z) = \frac{z^2+2az-3ab}{4z-a-3b},$$

Then the derivative;

$$N_3'(z) = \frac{(4z-a-3b)[2z+2a]-[(z^2+2az-3ab).4]}{(4z-a-3b)^2},$$

$$= \frac{8z^2+8az-2az-2a^2-6bz-6ab-4z^2-8az+12ab}{(4z-a-3b)^2},$$

$$= \frac{4z^2-2az-2a^2-6bz+6ab}{(4z-a-3b)^2},$$

$$N_3'(z) = \frac{(z-a)[4z+2a-6b]}{(4z-a-3b)^2},$$

The fixed point for N_3 is $z=a, z=b$, to find the critical point we put;

$$N_3'(a) = 0,$$

Then a the superattractive fixed point. $N_3'(b) = -2$ it is the neutral fixed point. Then the critical point there is;

$$N_3'(z) = 0,$$

$$\therefore\ N_3'(z) = \frac{(z-a)[4z+2a-6b]}{(4z-a-3b)^2},$$

$$\frac{(z-a)[4z+2a-6b]}{(4z-a-3b)^2} = 0,$$

$$(z-a)[4z+2a-6b] = 0,$$

Then;

$$(z-a)=0 \rightarrow z=a,$$

$$or\ [4z+2a-6b]=0,$$

$$z=\frac{3b-a}{2},$$

therefore, the basin of attraction of the triple root of $(z-a)^3(z-b)$ under N_3 is the whole of the complex plane, except a straight cut from $(3b-a)/2$ through b to ∞. For higher values of m, the Julia set is totally disconnected. Since the relaxed Newton's method applied to $g(z)$ is the same as the standard Newton's method applied to $\sqrt[m]{g(z)}$.

REFERENCES

[1] F. V. Haeseler and H.O. Peitgen, Newton's Method and Complex Dynamical Systems, Act Appl. Math. Volume 13, Pages 3 -58. 1988;

[2] William J. Gilbert, The complex dynamics of newton's method for a double root. Math. Appli, 22(10): 115-119, 1991.

[3] H. E. Benzinger, S. A. Burins and J. I. Palmore, Chaotic Complex Dynamics and Newton's Method, Phys. Lett. Volume 119, Pages 441-446 ,1987.

[4] P. Blanchard, Complex Analytic dynamics on the Riemann Sphere, Ball. Amer. Math. Soc. Volume 11, Pages 85-141 ,1984.

[5] R.L. Devaney, An Introduction to Chaotic Dynamical Systems, Addison-Wesley, Reading, Mass,1989.

معامل التأثير العربي 1.5
العدد 20
مجـلة الـتربـوي
Journal of Educational
ISSN: 2011- 421X
Arcif Q3

الفهـــــــــرس

مجلـة الــتربــوي
Journal of Educational
ISSN: 2011- 421X
Arcif Q3

معامل التأثير العربي 1.5
العدد 20

	مجـلة الـتربـوي Journal of Educational ISSN: 2011- 421X Arcif Q3	معامل التأثير العربي 1.5 العدد 20