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# مجلة التربوي

## مجلة علمية محكمة تصدر عن كلية التربية

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العدد العشرون  
يناير 2022م

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## Conjugate Newton's Method for a Polynomial of degree $m+1$

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### 0.1 Abstract

The main problem of this paper is to conjugate the cubic equation to quadratic equation by using Möbius transformation, so the work to finding the roots of polynomials with Complex variable and double roots using Newton's method will be easier. Also we make some correction to the linear fractional transformation (or Mobius transformation) which mentioned in [12].

### 0.2 Introduction

In this paper is devoted to investigate the using of the complex dynamic Newton's Method for a double Root. We discuss different cases including: simple root and multiple roots. To approximating the real or complex roots of a polynomial function  $g(z)$ , Newton's Method which consists of iterating the function  $N(z)$  is needed where;

$$N(z) = z - \frac{g(z)}{g'(z)}. \quad (0.2.1)$$

If we chose the , initial value for  $z_0$  to be sufficiently close to a root of  $g$ , the sequence of iterations ,  $z_{k+1} = N(z_k)$  will converge to the root. The type of convergence depends on the complexity of the root, if the root is simple, the sequence converges quadratically, but if the root is a multiple, the convergence can be only linear. Newton's method can be adjusted to have a better convergence to multiple roots. If the function  $g(z)$  has a multiple root of order exactly  $m$ , the applying Newton's method to  $\sqrt[m]{g(z)}$  leads to



$$N_m(z) = z - \frac{\sqrt[m]{g(z)}}{(\sqrt[m]{g(z)})'},$$
$$N_m(z) = z - \frac{(g(z))^{\frac{1}{m}}}{\frac{1}{m}g(z)^{\frac{1}{m}-1}g'(z)},$$
$$= \frac{[(g(z))^{\frac{1}{m}-1} \cdot g'(z)]z - m(g(z))^{\frac{1}{m}}}{g(z)^{\frac{1}{m}-1} \cdot g'(z)},$$

we get;

$$N_m(z) = \frac{(g(z))^{\frac{1}{m}-1} [g'(z)z - mg(z)]}{(g(z))^{\frac{1}{m}-1} \cdot g'(z)},$$
$$N_m(z) = \frac{g'(z)z - mg(z)}{g'(z)} = z - \frac{mg(z)}{g'(z)}, \quad (0.2.2)$$

This method is called relaxed Newton's method or Newton's method for a root of order  $m$ . This method converge quadratically to a root of order  $m$ .

Newton's method defines a dynamical system on the complex Riemann sphere for each polynomial function  $g(z)$ . Peitgen and Haeseler provide with an overview of the complex dynamics of Newton's method for a rational function, and discuss the basins of attraction of the roots for the relaxed Newton's method. We introduced how the relaxed Newton's method for a double root is applied to a cubic with a double root, and illustrate that the dynamics is conjugate to that of a well-known Julia set. Applying the relaxed Newton's method for a root of order  $m$ . to a polynomial of degree  $m + 1$ . Came up with a similar result. Also, it is shown that the Julia sets came by Bums, Palmore and Benzinger applying the standard Newton's method to the family of functions  $(z - 1)(z + \alpha)^k$  are conjugate to the Julia sets of quadratics.



### 0.3 Conjugate the Map $N$ by Transformation to obtain a Quadratic Polynomial

#### Theorem 1.

For the rational map  $N_p(z)$  arising from Newton's method applied to a quadratic polynomial  $p(z) = (z - a)(z - b)$ ,  $a \neq b$ ,  $N_p(z)$  is conjugate to  $z^2$  by the Möbius transformation.

#### Proof:

By substitution in Eq. (0.2.1) we obtaining

$$\begin{aligned} N(z) &= z - \frac{(z - a)(z - b)}{(z - a) + (z - b)}, \\ &= \frac{z[(z - a) + (z - b)] - (z - a)(z - b)}{(z - a) + (z - b)}, \\ &= \frac{z^2 - az + z^2 - bz - z^2 + bz + az - ab}{2z - a - b}, \\ N(z) &= \frac{z^2 - ab}{2z - a - b}, \end{aligned} \quad (0.3.1)$$

by derivative Eq. (0.3.1) we get

$$\begin{aligned} N'(z) &= \frac{(2z - a - b).2z - (z^2 - ab).2}{(2z - a - b)^2}, \\ &= \frac{4z^2 - 2az - 2bz - 2z^2 + 2ab}{(2z - a - b)^2}, \\ &= \frac{2z^2 - 2az - 2bz + 2ab}{(2z - a - b)^2}, \\ N'(z) &= \frac{2(z^2 - az - bz + ab)}{(2z - a - b)^2}, \end{aligned}$$

The fixed point of  $N$  are  $z = a$  and  $z = b$ , since  $N'(a) = N'(b) = 0$  then  $a$  and  $b$  are a super attractive fixed points.



The critical point of  $N$  are  $z = a$  and  $z = b$ .

The critical points of the quadratic  $p(z) = z^2 + c$  on the Riemann sphere are  $z = 0$  and  $z = \infty$  is also a super attractive fixed point.

the linear fractional transformation (or Möbius transformation). Using the transformation

$$h(z) = \frac{z - p}{z - q}, \quad (0.3.2)$$

Where  $p$  is turns to zero and  $q$  is turns to  $\infty$ .

Now we have  $h(z) = \frac{z-a}{z-b}$ ,

to find  $h^{-1}(z)$ , assume that

$$\begin{aligned} h^{-1}(z) = w &\rightarrow z = h(w), \\ z &= \frac{w - a}{w - b}, \\ \Rightarrow (w - b)z &= (w - a), \\ \Rightarrow wz - bz &= w - a, \\ wz - w &= bz - a, \\ w(z - 1) &= bz - a, \\ w = \frac{(bz - a)}{z - 1} &\Rightarrow h^{-1}(z) = \frac{bz - a}{z - 1}, \end{aligned}$$

Conjugate the map  $N$ , by transformation  $h$ , to obtain the quadratic;

Let ;  $p(z) = h \circ N \circ h^{-1}(z)$ ,

$$\begin{aligned} N \circ h^{-1}(z) &= \frac{z^2 - ab}{2z - a - b} \circ \frac{bz - a}{z - 1} = \frac{\left(\frac{bz - a}{z - 1}\right)^2 - ab}{2\left(\frac{bz - a}{z - 1}\right) - a - b}, \\ &= \frac{[(bz - a)^2 - ab(z - 1)^2]}{[2(bz - a)(z - 1) - (a + b)(z - 1)^2]}, \end{aligned}$$



$$\begin{aligned} &= \frac{[b^2z^2 - 2abz + a^2 - abz^2 + 2abz - ab]}{[2bz^2 - 2bz - 2az + 2a - az^2 + 2az - a - bz^2 + 2bz - b]}, \\ &= \frac{b^2z^2 + a^2 - abz^2 - ab}{bz^2 + a - az^2 - b}, \\ &= \frac{(bz^2 - a)[b - a]}{(z^2 - 1)[b - a]} = \frac{bz^2 - a}{z^2 - 1}, \end{aligned}$$

therefore

$$\begin{aligned} h \circ N \circ h^{-1}(z) &= \frac{z - a}{z - 1} \circ \frac{bz^2 - a}{z^2 - 1}, \\ \Rightarrow \frac{\frac{bz^2 - a}{z^2 - 1} - a}{\frac{bz^2 - a}{z^2 - 1} - b} &= \frac{bz^2 - a - a(z^2 - 1)}{bz^2 - a - b(z^2 - 1)}, \\ &= \frac{bz^2 - a - az^2 + a}{bz^2 - a - bz^2 + b} = \frac{z^2(b - a)}{b - a} = z^2. \end{aligned}$$

## Theorem 2.

The relaxed Newton's method  $N_2$  for a double root applied to any cubic equation with a double root is conjugate by a linear fractional transformation on the Riemann sphere to the iterations of the quadratic

$$p(z) = \frac{3z^2 - 1}{2}$$

### Proof:

Assume that cubic be  $g(z) = (z - a)^2(z - b)$ , where  $a$  and  $b$  are distinct complex numbers. By applying the Newton's method for a double root to a this cubic, we obtain the following;

$$\begin{aligned} N_2(z) &= z - \frac{2g(z)}{g'(z)}, \\ N_2(z) &= z - \frac{2[(z - a)^2(z - b)]}{[(z - a)^2(z - b)]'}, \end{aligned}$$



$$\begin{aligned} &= z - \frac{2[(z-a)(z-a)(z-b)]}{[(z-a)^2 \cdot 1 + (z-b) \cdot 2(z-a) \cdot 1]}, \\ &= z - \frac{(z-a)[3(z-a)(z-b)]}{(z-a)[(z-a) + 2(z-b)]}, \\ &= z - \frac{2[(z-a)(z-b)]}{[(z-a) + 2(z-b)]}, \\ &= \frac{z^2 - az + 2z^2 - 2bz - 2z^2 + 2bz + 2az - 2ab}{z - a + 2z - 2b}, \\ N_2(z) &= \frac{z^2 + az - 2ab}{3z - a - 2b}, \end{aligned} \quad (0.3.3)$$

by derivative Eq. (0.3.3) we obtaining

$$\begin{aligned} N_2'(z) &= \frac{(3z - a - 2b)[2z + a] - [(z^2 + az - 2ab) \cdot 3]}{(3z - a - 2b)^2}, \\ &= \frac{6z^2 + 3az - 2az - a^2 - 4bz - 2ab - 3z^2 - 3az + 6ab}{(3z - a - 2b)^2}, \\ &= \frac{3z^2 - 2az - a^2 - 4bz + 4ab}{(3z - a - 2b)^2}, \\ N_2'(z) &= \frac{(z-a)[3z + a - 4b]}{(3z - a - 2b)^2}. \end{aligned} \quad (0.3.4)$$

It is clear that  $N_2$  has fixed points of  $z = a$  and  $z = b$ . when  $z = a$  the derivative  $N_2'(a) = 0$ , So,  $a$  is a super attractive fixed point, and when

$z = b$  the  $N_2'(b) = -1$ , then  $b$  is a neutral fixed point.

The critical points of  $N_2'(z) = 0$ ,

To find critical points, we put  $N_2'(z) = 0$  in Eq. (0.3.4)

$$\frac{(z-a)(3z+a-4b)}{(3z-a-2b)^2} = 0 \rightarrow (z-a)(3z+a-4b) = 0,$$

$$z - a = 0, \Rightarrow z = a$$





$$\text{or } 3z + a - 4b = 0,$$

$$3z = 4b - a, \quad \Rightarrow z = \frac{4b - a}{3},$$

The critical points of the quadratic  $p(z) = z^2 + c$  on the Riemann sphere;

$$p'(z) = 2z,$$

To find critical point we put  $p'(z) = 0$ ;

$$2z = 0 \Rightarrow z = 0, \quad \text{or } z = \infty,$$

Since  $z = \infty$  is a super attractive fixed point too.

Mobius transformation on the linear fractional transformation, by using transformation in Eq. (0.3.2)

where  $p$  is to be transform to zero, and  $q$  is to be transform to  $\infty$ ,

$$\begin{aligned} h(z) &= \frac{z - \left(\frac{4b - a}{3}\right)}{z - a}, \\ &= \frac{3z - 4b + a}{3z - 3a}, \\ h(z) &= \frac{3z - 4b + a}{3(z - a)}. \end{aligned}$$

to find  $h^{-1}(z)$ , we suppose that;

$$h^{-1}(z) = w \rightarrow z = h(w),$$

$$z = \frac{3w + a - 4b}{3(w - a)},$$

$$3z(w - a) = 3w + a - 4b,$$

$$3zw - 3za = 3w + a - 4b,$$

$$3zw - 3w = 3az + a - 4b,$$

$$w(3z - 3) = 3az + a - 4b,$$



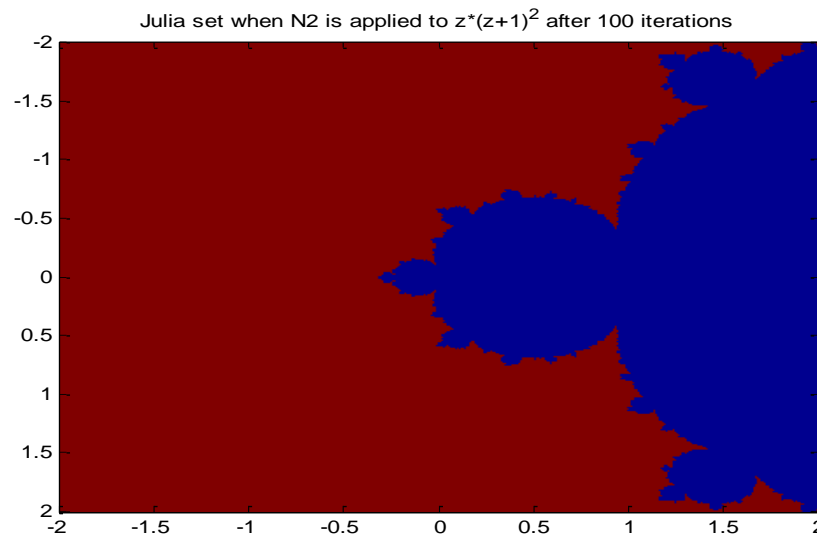
$$h^{-1}(z) = w = \frac{3az + a - 4b}{3z - 3},$$

Sends  $a$  to  $\infty$  and  $(4b - a)/3$  to 0, and map the critical points of  $N_2$  to the critical points of the quadratic  $p$ . use the transformation  $h$  to Conjugate the map  $N_2$ , by the transformation  $h$ , to obtain the quadratic, using MATLAB the result could be obtained as:

$$p(z) = h \circ N_2 \circ h^{-1}(z) = \frac{3z^2 - 1}{2}.$$

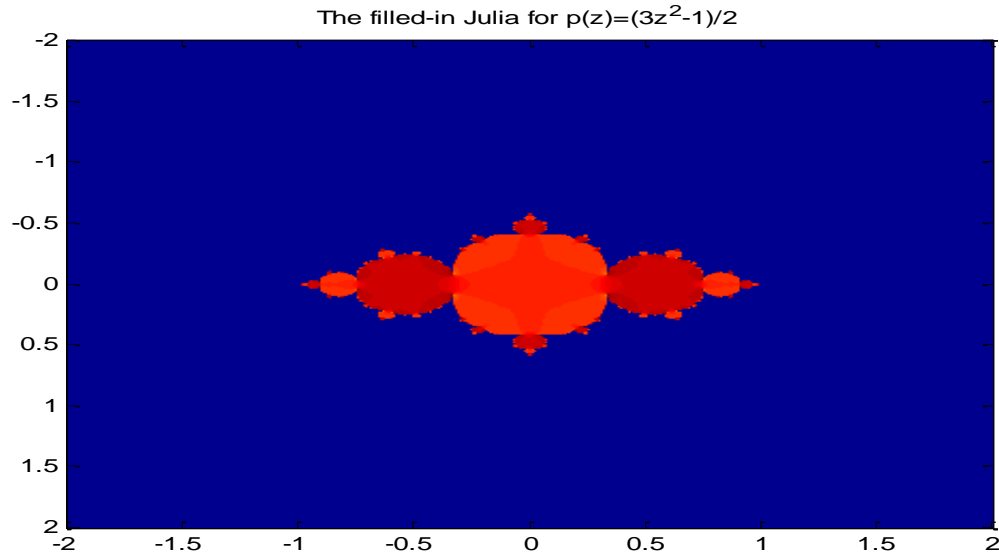
We chose The constant factor of  $h$  was chosen so that the coefficient of  $z^2$  became  $3/2$ .

Therefore , we found that under the map  $h$  on the Riemann sphere, the dynamics of the relaxed Newton's method  $N_2$  is conjugate to the dynamics of  $p$ .

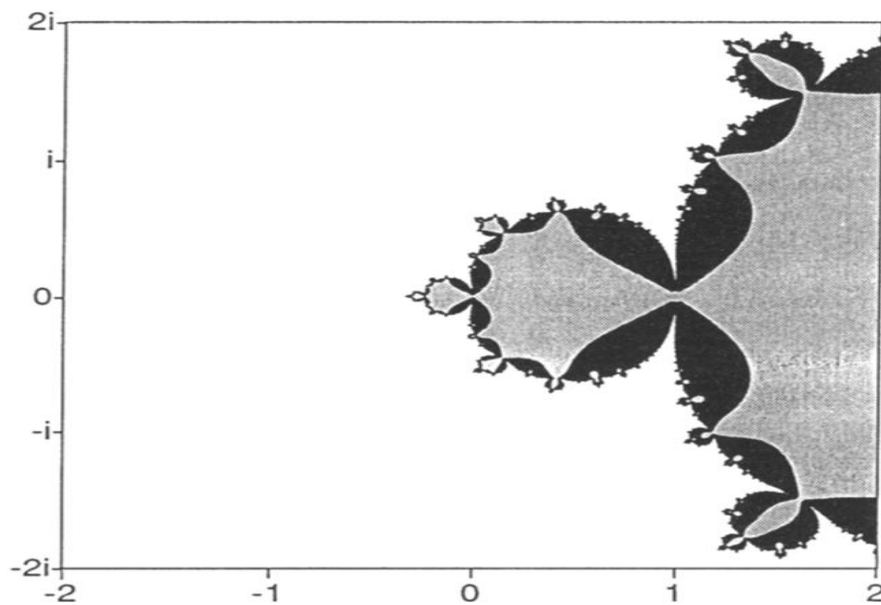


**Figure 1:** the Brown area is the basin of attraction, after 100 iterations, of the double root  $-1$ , when  $N_2$  is applied to  $(z + 1)^2 z$ .

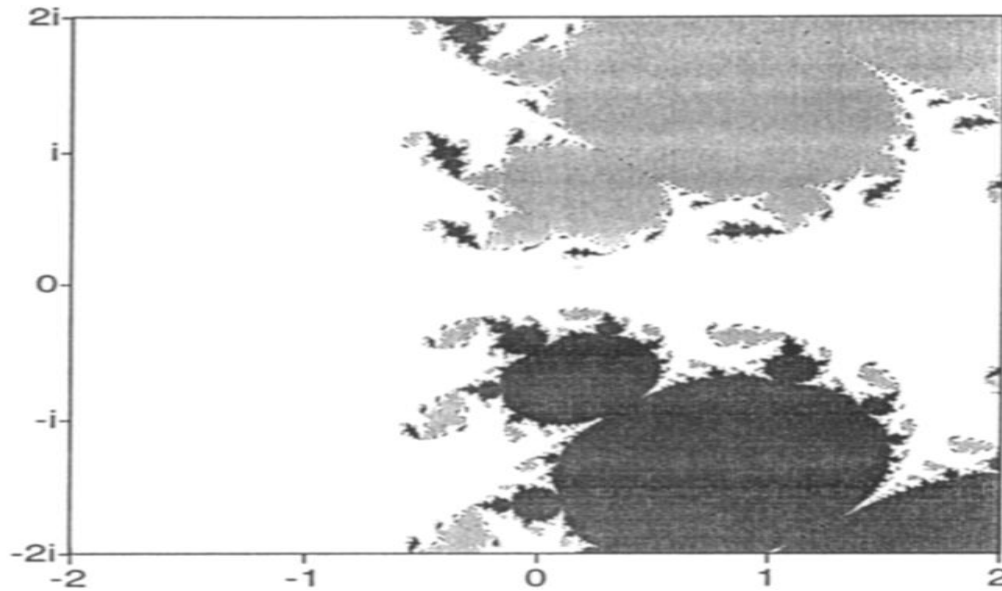
The previous Figure 1 illustrate the basin of attraction for the double root in the complex plane when the relaxed Newton's method  $N_2$  is applied to the cubic  $g(z) = z(z + 1)^2$ . All initial values in the brown region will converge to the double root  $a = -1$ , while initial values the blue region do not converge to within 0.001 of a root in 100 iterations.



**Figure 2:** The filled-in Julia set for  $p(z) = (3z^2 - 1)/2$



**Figure 3:** The same as Figure 1, except after 1195 iterations points in the gray area come to within 0.01 of the simple root 0.



**Figure 4:** the basins of attraction for  $N_2$  applied to  $p(z) = (z + 1)^2(z^2 + 0.25)$ . After 1000 iterations the point shaded white, light gray and dark gray, come to within 0.01 of the roots  $-1$ ,  $i/2$  and  $-i/2$ , respectively.

Hence the points inside the blue region in figure 1 do converge to the root  $b = 0$ , but do so extremely slowly, as shown in figure 3. However, there is no complete neighborhood of  $b$  in which points all converge to  $b$ , since any such neighborhood intersects the Julia set. Points starting on the Julia set will not converge, but will remain on the boundary under iterations of  $N_2$ .

Benzinger, Burns, and Palmore discuss Newton's method for the family of functions;

$$f_{\alpha}(z) = (z + \alpha)^{\alpha}(z - 1),$$

For  $\alpha = 1/2$ , this gives the function  $(z - 1)\sqrt{z + 0.5}$  and the basins of attraction for Newton's method, this is as the same our figure 1, since it follows from equation (0.2.2) that the relaxed Newton's method,  $N_2$ , for  $(z - 1)^2(z + 0.5)$  is the same as Newton's method,  $N$  for  $(z - 1)\sqrt{z + 0.5}$ .



#### 0.4 Conjugate polynomial of degree $m + 1$ and using Newton's method to find the roots

To investigate more generally what happens to the relaxed Newton's method, apply  $N_m$  to the function  $g(z) = q(z)(z - b)^k$ . Assume that  $b$  is not a root of  $q$  and assume that  $q$  has finite derivative. The root  $b$  has order  $k$  and;

$$N_m(z) = z - \frac{mg(z)}{g'(z)},$$

$$g(z) = q(z)(z - b)^k,$$

$$N_m(z) = z - \frac{m(q(z)(z - b)^k)}{q(z) \cdot k(z - b)^{k-1} + (z - b)^k q'(z)},$$

Then we get;

$$N_m(z) = z - \frac{mq(z)(z - b)^{k-1} \cdot (z - b)}{(z - b)^{k-1} [kq(z) + (z - b)q'(z)]},$$

$$N_m(z) = z - \frac{mq(z) \cdot (z - b)}{kq(z) + (z - b)q'(z)},$$

New, we find  $N'_m(z)$ ;

$$N'_m(z) = 1 - \frac{[kq(z) + (z - b)q'(z)]m[kq(z) + (z - b)q'(z)]}{[kq(z) + (z - b)q'(z)]^2} - \frac{m[q(z)(z - b) \cdot kq'(z) + (z - b)''q(z) \cdot q'(z)]}{[kq(z) + (z - b)q'(z)]^2},$$

$$N'_m(b) = 1 - \frac{mkq^2(b)}{k^2q(b)} \Rightarrow N'_m(b) = 1 - \frac{m}{k}.$$

So that  $b$  is a super attractive fixed point only if  $k = m$ . Otherwise, if  $k < \frac{m}{2}$ ,  $|N'_m(b)| > 1$  and  $b$  is a repulsive fixed point. If  $k = \frac{m}{2}$ ,  $|N'_m(b)| = 1$  and  $b$  is a neutral fixed point, as the cubic above shows. If  $k > \frac{m}{2}$ ,  $|N'_m(b)| < 1$  and  $b$  is an attractive fixed point for  $N_m$ , but the convergence is only linear, not quadratic.

In the typical situation where we apply the relaxed Newton's method,  $N_2$  to a function with one double root and no other multiple roots, all the simple roots will



be neutral fixed points. For example, figure 4 shown the basins of attraction for a quartic with one double root. Theorem (3) and its proof can be easily generalized to the relaxed Newton's method  $N_m$ .

### Theorem 3.

The relaxed Newton's method  $N_m$  applied to any polynomial of degree  $(m + 1)$  with a root of order  $m$ , is conjugate by a linear fractional transformation on the Riemann sphere, to the iterations of the quadratic;

Proof:

$$\begin{aligned} \because g(z) &= (z - a)^m(z - b), \\ N_m(z) &= z - \frac{mg(z)}{g'(z)}, \\ N_m(z) &= z - \frac{m[(z - a)(z - b)]}{(z - a)^m + m(z - a)^{m-1}(z - b)}, \\ &= z - \frac{m[(z - a)(z - b)]}{(z - a)^{m-1}[(z - a) + m(z - b)]}, \\ &= z - \frac{m[(z - a)(z - b)]}{(z - a) + m(z - b)}, \\ &= \frac{z^2 - az + amz - abm}{z - a + mz - mb}, \end{aligned} \quad (0.4.1)$$

by derivative Eq. (0.4.1), we get

$$\begin{aligned} N'_m(z) &= \frac{(z - a + mz - bm)[2z - a + am]}{(z - a + mz - mb)^2} \\ &\quad - \frac{[(z^2 - az + amz - abm)(1 + m)]}{(z - a + mz - mb)^2}, \\ &= \frac{z^2 - 2az + a^2 - a^2m + mz^2 - 2bmz + 2abm}{(z - a + mz - mb)^2}, \end{aligned}$$

If we used the long division we find  $z = a$  it be exercise the numerator, then we get;



$$N'_m(z) = \frac{(z - a)[(z - a + am - 2bm + mz)]}{(z - a + mz - mb)^2},$$

After that, we find the critical points for  $N'_m$

$$\therefore N'_m(z) = \frac{(z - a)[(z - a + am - 2bm + mz)]}{(a - z + bm - mz^2)},$$

We put  $N'_m = 0$

$$(z - a)[(z - a + am - 2bm + mz)] = 0,$$

$$(z - a) = 0 \Rightarrow z = a,$$

$$\text{or } [(z - a + am - 2bm + mz)] = 0,$$

$$z + mz = a - am + 2bm,$$

$$z = \frac{a - am + 2bm}{1 + m}, \quad m \neq -1,$$

Now, we use the transformation

$$h(z) = \frac{z - p}{z - q},$$

where  $p$  is to be transform to zero, and  $q$  is to be transform to  $\infty$ ,

$$\begin{aligned} h_m(z) &= \frac{z - \left(\frac{a - am + 2bm}{1 + m}\right)}{z - a}, \\ h_m(z) &= \frac{\frac{z + mz - a + am - 2bm}{1 + m}}{z - a}, \\ &= \frac{z + mz - a + am - 2bm}{(1 + m)(z - a)}, \end{aligned}$$

Then, we find  $h_m^{-1}$ ;

$$\begin{aligned} h_m^{-1}(z) &= w \rightarrow z = h_m(w), \\ z &= \frac{w + mw - a + am - 2bm}{(1 + m)(w - a)}, \end{aligned}$$



$$\begin{aligned}z(1+m)(w-a) &= w + mw - a + am - 2bm, \\wz - az + mzw - amz &= w + mw - a + am - 2bm, \\wz + mwz - w - mw &= az + amz - a + am - 2bm, \\w(z + mz - 1 - m) &= az + amz - a + am - 2bm, \\w &= \frac{az + amz - a - am - 2bm}{(z + mz - 1 - m)} \\&= \frac{az + amz - a - am - 2bm}{(z-1) + m(z-1)}, \\w &= \frac{az + amz - a - am - 2bm}{(z-1)(1+m)} = h_m^{-1}(z),\end{aligned}$$

And by using the MATLAB, the result could be obtained as:

$$p(z) = h \circ N_m \circ h^{-1} = \frac{(mz^2 - m + z^2 + 1)}{2},$$

In particular, when  $m = 3$ , the Julia set of  $2z^2 - 1$  is the line segment on the real axis between -1 and 1. Then;

$$\begin{aligned}p(z) &= (z-a)^3(z-b), \\N_3(z) &= z - \frac{3g(z)}{g'(z)}, \\N_3(z) &= z - \frac{3[(z-a)^3(z-b)]}{[(z-a)^3(z-b)]'}, \\&= z - \frac{3[(z-a)(z-a)(z-a)(z-b)]}{[(z-a)^3 \cdot 1 + (z-b) \cdot 3(z-a)^2 \cdot 1]}, \\&= z - \frac{3[(z-a)^2(z-a)(z-b)]}{(z-a)^2[(z-a) + 3(z-b)]}, \\&= z - \frac{3(z-a)^2[(z-a)(z-b)]}{(z-a)^2[(z-a) + 3(z-b)]},\end{aligned}$$





$$\begin{aligned} &= z - \frac{3[(z-a)(z-b)]}{z-a+3z-3b}, \\ &= z - \frac{3[z^2 - bz - az + ab]}{4z - a - 3b}, \\ &= \frac{4z^2 - az - 3bz - 3z^2 + 3bz + 3az - 3ab}{4z - a - 3b}, \\ N_3(z) &= \frac{z^2 + 2az - 3ab}{4z - a - 3b}, \end{aligned}$$

Then the derivative;

$$\begin{aligned} N_3'(z) &= \frac{(4z - a - 3b)[2z + 2a] - [(z^2 + 2az - 3ab).4]}{(4z - a - 3b)^2}, \\ &= \frac{8z^2 + 8az - 2az - 2a^2 - 6bz - 6ab - 4z^2 - 8az + 12ab}{(4z - a - 3b)^2}, \\ &= \frac{4z^2 - 2az - 2a^2 - 6bz + 6ab}{(4z - a - 3b)^2}, \\ N_3'(z) &= \frac{(z - a)[4z + 2a - 6b]}{(4z - a - 3b)^2}, \end{aligned}$$

The fixed point for  $N_3$  is  $z = a, z = b$ , to find the critical point we put;

$$N_3'(a) = 0,$$

Then  $a$  the superattractive fixed point.  $N_3'(b) = -2$  it is the neutral fixed point. Then the critical point there is;

$$\begin{aligned} N_3'(z) &= 0, \\ \therefore N_3'(z) &= \frac{(z - a)[4z + 2a - 6b]}{(4z - a - 3b)^2}, \\ \frac{(z - a)[4z + 2a - 6b]}{(4z - a - 3b)^2} &= 0, \\ (z - a)[4z + 2a - 6b] &= 0, \end{aligned}$$



Then;

$$(z - a) = 0 \rightarrow z = a,$$

$$\text{or } [4z + 2a - 6b] = 0,$$

$$z = \frac{3b - a}{2},$$

therefore, the basin of attraction of the triple root of  $(z - a)^3(z - b)$  under  $N_3$  is the whole of the complex plane, except a straight cut from  $(3b - a)/2$  through  $b$  to  $\infty$ . For higher values of  $m$ , the Julia set is totally disconnected. Since the relaxed Newton's method applied to  $g(z)$  is the same as the standard Newton's method applied to  $\sqrt[m]{g(z)}$ .

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