



مجلة التربوي
Journal of Educational
ISSN: 2011- 421X
Arcif Q3

معامل التأثير العربي 1.5
العدد 20



مجلة التربوي

مجلة علمية محكمة تصدر عن كلية التربية

جامعة المرقب

العدد العشرون
يناير 2022م

هيئة تحرير
مجلة التربوي

- المجلة ترحب بما يرد عليها من أبحاث وعلى استعداد لنشرها بعد التحكيم .
 - المجلة تحترم كل الاحترام آراء المحكمين وتعمل بمقتضاها .
 - كافة الآراء والأفكار المنشورة تعبر عن آراء أصحابها ولا تتحمل المجلة تبعاتها .
 - يتحمل الباحث مسؤولية الأمانة العلمية وهو المسؤول عما ينشر له .
 - البحوث المقدمة للنشر لا ترد لأصحابها نشرت أو لم تنشر .
- (حقوق الطبع محفوظة للكلية)



ضوابط النشر:

- يشترط في البحوث العلمية المقدمة للنشر أن يراعى فيها ما يأتي :
- أصول البحث العلمي وقواعده .
 - ألا تكون المادة العلمية قد سبق نشرها أو كانت جزءا من رسالة علمية .
 - يرفق بالبحث تزكية لغوية وفق أنموذج معد .
 - تعدل البحوث المقبولة وتصحح وفق ما يراه المحكمون .
 - التزام الباحث بالضوابط التي وضعتها المجلة من عدد الصفحات ، ونوع الخط ورقمه ، والفترات الزمنية الممنوحة للتعديل ، وما يستجد من ضوابط تضعها المجلة مستقبلا .

تنبيهات :

- للمجلة الحق في تعديل البحث أو طلب تعديله أو رفضه .
- يخضع البحث في النشر لأولويات المجلة وسياستها .
- البحوث المنشورة تعبر عن وجهة نظر أصحابها ، ولا تعبر عن وجهة نظر المجلة .

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Sufficient Conditions of Bounded Radius Rotations for Two Integral Operators

Defined by q -Analogue of Ruscheweyh Operator

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Abstract

Motivating by q -analogue theory, in this article, we derive the q -analogue of Ruscheweyh differential Operator using q -hypergeometric function and using this new operator to define integral operators which generalizes many operators introduced before. We also consider subclasses of analytic functions with bounded radius and bounded boundary rotations and study the mapping properties of these classes under these q -integral operators.

1 Introduction

Let A be the class of all functions of the following form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k$$

Which are analytic in the open unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$. A function $f \in A$ is said to be spiral-like if there exists a real number λ ($|\lambda| < \pi/2$) such that

$$\operatorname{Re} \left\{ e^{i\lambda} \frac{zf'(z)}{f(z)} \right\} > 0, \quad (z \in U).$$

The class of all spiral-like functions has been introduced by Spacek [1] in 1933 and we denote it by S_λ^* . Later in 1969, Robertson [2] considered the class C_λ of analytic functions in U for which $zf'(z) \in S_\lambda^*$.

Let $P_k^\lambda(\xi)$ be the class of functions $p(z)$ analytic in U with $p(0) = 1$ and



$$\int_0^{2\pi} \left| \frac{\operatorname{Re} e^{i\lambda} p(z) - \xi \cos \lambda}{1 - \xi} \right| d\theta \leq k\pi \cos \lambda, \quad z = re^{i\theta},$$

where $k \geq 2$, $0 \leq \xi < 1$, λ is real with $|\lambda| < \pi/2$.

For $\lambda = 0$, this class was introduced in [3] and for $\xi = 0$, refer to [4]. For $k = 2$, $\lambda = 0$ and $\xi = 0$, the class $P_k^\lambda(\xi)$ reduces to the class P of functions $p(z)$ analytic in U with $p(0)=1$ and whose real part is positive. For $k < 2$, the functions in p_k may not have positive real part.

One of the most important summation formulas for hypergeometric series is given by the binomial theorem:

$$\begin{aligned} {}_2F_1(a, c, c; z) &= {}_1F_0(a, -; z) \\ &= \sum_{k=0}^{\infty} \frac{(a)_k}{k!} z^k = (1-z)^{-a}, \quad a \in \mathbb{R}, \end{aligned}$$

where ${}_2F_1$ denotes Gaussian hypergeometric function and $|z| < 1$. The q -analogue of this formula is defined by

$${}_1\phi_0(a, -; q; z) = \sum_{k=0}^{\infty} \frac{(a; q)_k}{(q; q)_k}, \quad |z| < 1, |q| < 1, \quad (1.1)$$

which was derived by Cauchy (1843), Heine (1847) and by other mathematicians. $(a, q)_k$ is the q -analogue of the Pochhammer symbol $(a)_k$ defined by

$$(a, q)_k = \begin{cases} 1, & k = 0: \\ (1-a)(1-aq)(1-aq^2) \dots (1-aq^{k-1}), & k \in \mathbb{N}. \end{cases}$$

It is clear that

$$\lim_{q \rightarrow 1} \frac{(q^a; q)_k}{(1-q)^k} = (a)_k.$$

By using the ratio test, one recognizes that, if $|q| < 1$, the q series (1.1) (also called basic hypergeometric series) converges absolutely for $|z| < 1$. For more details concerning the q -theory the reader may refer to ([5],[6]).



Replacing a by $(\delta + 1, \delta > -1)$ in (1.1), we now define the q -analogue of Ruscheweyh differential operator $D_q^\delta f : A \rightarrow A$ as follows:

$$D_q^\delta f(z) = z {}_1\phi_0(\delta + 1, -; q; z) * f(z)$$
$$= z + \sum_{k=2}^{\infty} \frac{(\delta+1; q)_{k-1}}{(q; q)_{k-1}} a_k z^k, \delta > -1, |z| < 1, |q| < 1. \quad (1.2)$$

Observe that if $\delta + 1 = q^{\mu+1}, \mu > -1$, we have

$$\lim_{q \rightarrow 1} D_q^\delta f(z) = z + \lim_{q \rightarrow 1} \left[\sum_{k=2}^{\infty} \frac{(q^{\mu+1}; q)_{k-1}}{(q; q)_{k-1}} a_k z^k \right]$$
$$= z + \sum_{k=2}^{\infty} \frac{(\mu + 1)_{k-1}}{(k-1)!} a_k z^k = D^\mu f(z),$$

where $D^\mu f(z)$ is Ruscheweyh differential operator defined in [7].

Next, we define a new subclasses of analytic functions of complex order involving the q -analogue of Ruscheweyh differential operator.

Definition 1.1 A function $f \in A$ is said to belong to the class $R_k^\lambda(\xi, b, \delta, q)$ if and only if

$$1 + \frac{1}{b} \left(\frac{z (D_q^\delta f(z))'}{D_q^\delta f(z)} - 1 \right) \in P_k^\lambda(\xi),$$

where $k \geq 2, 0 \leq \xi < 1, \lambda$ is real with $|\lambda| < \pi/2, b \in \mathbb{C} - \{0\}$.

Definition 1.2 A function $f \in A$ is said to belong to the class $V_k^\lambda(\xi, b, \delta, q)$ if and only if

$$1 + \frac{1}{b} \frac{z (D_q^\delta f(z))''}{(D_q^\delta f(z))'} \in P_k^\lambda(\xi),$$

where $k \geq 2, 0 \leq \xi < 1, \lambda$ is real with $|\lambda| < \pi/2, b \in \mathbb{C} - \{0\}$.

Remark 1.1 i) Letting $\delta = q - 1, b = 1$, we obtain the classes $R_k^\lambda(\xi), V_k^\lambda(\xi)$ respectively, introduced and studied by Noor et al. [8] and Moulis [9].



ii) For $\delta = q - 1$ and $\lambda = 0$, we have the classes $R_k(\xi, b), V_k(\xi, b)$, respectively, introduced and studied by Noor et al. [10].

iii) For $\delta = q - 1, k = 2$ and $\lambda = 0$, we have the classes $S_\xi^*(b), C_\xi(b)$, respectively introduced and studied by Frasion [11].

iv) For $\delta = q - 1, b = 1, \xi = 0$ and $\lambda = 0$, we obtain the well known classes R_k, V_k of analytic functions with bounded radius and bounded boundary rotations introduced and studied by Tammi [12] and Paatero [13].

Definition 1.3 Let $m \in \mathbb{N} \cup \{0\}, j \in \{1, 2, \dots, m\}$, and $\alpha_j > 0$. One defines the integral operator $I(f_1, f_2, \dots, f_m): A^m \rightarrow A$ as $I(f_1, \dots, f_m) = F$,

$$D_q^\delta F(z) = \int_0^z \left(\frac{D_q^\delta f_1(t)}{t} \right)^{\alpha_1} \dots \left(\frac{D_q^\delta f_m(t)}{t} \right)^{\alpha_m} dt \quad (z \in U), \quad (1.3)$$

where $f_j \in A$.

Remark 1.2 The integral operator $D_q^\delta F$ generalizes many operators which were introduced and studied recently.

i) For $\delta + 1 = q^{\mu+1}, \mu > -1$, and $q \rightarrow 1$, we obtain the integral operator

$$D^\mu F(z) = \int_0^z \left(\frac{D^\mu f_1(t)}{t} \right)^{\alpha_1} \dots \left(\frac{D^\mu f_m(t)}{t} \right)^{\alpha_m} dt, \quad (1.4)$$

where D^μ is the Ruscheweyh differential operator. The integral operator $D^\mu F(z)$ introduced and studied by G. Oros et al. [14].

ii) For $\delta = q - 1, D_q^{q-1} f_j(z) = f_j \in A, j \in \{1, 2, \dots, m\}$, we have the integral operator

$$F_m(z) = \int_0^z \left(\frac{f_1(t)}{t} \right)^{\alpha_1} \dots \left(\frac{f_m(t)}{t} \right)^{\alpha_m} dt \quad (1.5)$$

introduced by D. Breaz and N. Breaz [15].



iii) For $\delta = q - 1, m = 1, \alpha_1 = \alpha \in [0,1], \alpha_2 = \dots = \alpha_m = 0$ and $D_q^{q-1} f_1 = f \in S^*$, (consists of functions that are analytic, univalent and starlike), we have the integral operator

$$F_\alpha(z) = \int_0^z \left(\frac{f(t)}{t} \right)^\alpha dt \quad (1.6)$$

introduced by Miller et. al. [16].

iv) For $\delta = q - 1, m = 1, \alpha_1 = 1, \alpha_2 = \dots = \alpha_m = 0$ and $D_q^{q-1} f_1 = f \in A$, we have the integral operator

$$F(z) = \int_0^z \frac{f(t)}{t} dt \quad (1.7)$$

introduced by Alexander. [17].

Definition 1.4 Let $m \in \mathbb{N}_0, j \in \{1, 2, \dots, m\}$, and $\alpha_j > 0$. One defines the integral operator $I(f_1, f_2, \dots, f_m): A^m \rightarrow A$ as $I(f_1, \dots, f_m) = H$,

$$D_q^\delta H(z) = \int_0^z \left[\left(D_q^\delta f_1(t) \right)' \right]^{\alpha_1} \dots \left[\left(D_q^\delta f_m(t) \right)' \right]^{\alpha_m} dt \quad (z \in U), \quad (1.8)$$

where $f_j \in A$.

Remark 1.3 The integral operator $D_q^\delta H$ generalizes many operators which were introduced and studied recently.

i) For $\delta = q - 1$ and $D_q^{q-1} f_j = f_j \in A, j \in \{1, 2, \dots, m\}$, we have the integral operator

$$H(z) = \int_0^z (f_1'(t))^{\alpha_1} \dots (f_m'(t))^{\alpha_m} dt \quad (1.9)$$

introduced by Breaz et. al. [18].

ii) For $\delta = q - 1, m = 1, \alpha_1 = \alpha \in \mathbb{C}, \alpha_2 = \dots = \alpha_m = 0$ and $D_q^{q-1} f_1 = f \in A$, we have the integral operator



$$H_\alpha(z) = \int_0^z (f_1'(t))^\alpha dt \quad (1.10)$$

introduced by Pfaltzgraff [19] (see also Pascu and pascar [20]).

In this paper, we investigate some properties of the above integral operators $D_q^\delta F$ and $D_q^\delta H$ for the classes $R_k^\lambda(\xi_j, b, \delta, q)$ and $V_k^\lambda(\xi_j, b, \delta, q)$.

2 Main results

Theorem 2.1 Let $f_j \in R_k^\lambda(\xi_j, b, \delta, q)$, for $j \in \{1, 2, \dots, m\}$ with $0 \leq \xi_j < 1, b \in \mathbb{C} - \{0\}$. Also let λ be real with $|\lambda| < \pi/2, \alpha_j > 0, j \in \{1, 2, \dots, m\}$. If

$$0 \leq 1 + \sum_{j=1}^m \alpha_j (\xi_j - 1) < 1,$$

then the integral operator F defined by (1.3) is in the class $V_k^\lambda(\gamma, b, \delta, q)$ with

$$\gamma = 1 + \sum_{j=1}^m \alpha_j (\xi_j - 1) \quad (2.1)$$

Proof. Since $f_j \in A, j \in \{1, 2, \dots, m\}$, by (1.2), we have

$$\frac{D_q^\delta f_j(z)}{z} = 1 + \sum_{k=2}^{\infty} \frac{(\delta+1; q)_{k-1}}{(q; q)_{k-1}} \alpha_{kj} z^{k-1} \neq 0 \quad \text{for all } z \in U.$$

By (1.3), we get

$$\left(D_q^\delta F(z) \right)' = \left(\frac{D_q^\delta f_1(z)}{z} \right)^{\alpha_1} \dots \left(\frac{D_q^\delta f_m(z)}{z} \right)^{\alpha_m}.$$

This equality implies that



$$\ln \left(D_q^\delta F(z) \right)' = \alpha_1 [\ln D_q^\delta f_1(z) - \ln z] + \dots + \alpha_m [\ln D_q^\delta f_m(z) - \ln z].$$

By differentiating the last equality, we have

$$\frac{\left(D_q^\delta F(z) \right)''}{\left(D_q^\delta F(z) \right)'} = \sum_{j=1}^m \alpha_j \left(\frac{\left(D_q^\delta f_j(z) \right)'}{D_q^\delta f_j(z)} - \frac{1}{z} \right).$$

Multiplying with z and $1/b$ both sides of the last relation we obtain

$$\begin{aligned} \frac{1}{b} \frac{z \left(D_q^\delta F(z) \right)''}{\left(D_q^\delta F(z) \right)'} &= \sum_{j=1}^m \alpha_j \frac{1}{b} \left(\frac{z \left(D_q^\delta f_j(z) \right)'}{D_q^\delta f_j(z)} - 1 \right) \\ &= \sum_{j=1}^m \alpha_j \left[1 + \frac{1}{b} \left(\frac{z \left(D_q^\delta f_j(z) \right)'}{D_q^\delta f_j(z)} - 1 \right) \right] - \sum_{j=1}^m \alpha_j, \end{aligned}$$

or equivalently

$$e^{i\lambda} \left(1 + \frac{1}{b} \frac{z \left(D_q^\delta F(z) \right)''}{\left(D_q^\delta F(z) \right)'} \right) = \left(1 - \sum_{j=1}^{\infty} \alpha_j \right) e^{i\lambda} + \sum_{j=1}^{\infty} \alpha_j e^{i\lambda} \left[1 + \frac{1}{b} \left(\frac{z \left(D_q^\delta f_j(z) \right)'}{D_q^\delta f_j(z)} - 1 \right) \right].$$

Subtracting and adding $\cos \lambda \sum_{j=1}^{\infty} \alpha_j \xi_j$ on the left hand side and taking the real part, we obtain

$$\operatorname{Re} \left\{ e^{i\lambda} \left(1 + \frac{1}{b} \frac{z \left(D_q^\delta F(z) \right)''}{\left(D_q^\delta F(z) \right)'} \right) - \gamma \cos \lambda \right\} = \sum_{j=1}^{\infty} \alpha_j \operatorname{Re} \left\{ e^{i\lambda} \left[1 + \frac{1}{b} \left(\frac{z \left(D_q^\delta f_j(z) \right)'}{D_q^\delta f_j(z)} - 1 \right) \right] - \xi_j \cos \lambda \right\},$$

where γ is given by (2.1). Integrating the last equation, we have

$$\int_0^{2\pi} \left| \operatorname{Re} \left\{ e^{i\lambda} \left(1 + \frac{1}{b} \frac{z \left(D_q^\delta F(z) \right)''}{\left(D_q^\delta F(z) \right)'} \right) - \gamma \cos \lambda \right\} \right| d\theta$$



$$\leq \sum_{j=1}^{\infty} \alpha_j \int_0^{2\pi} \left| \operatorname{Re} \left\{ e^{i\lambda} \left[1 + \frac{1}{b} \left(\frac{z (D_q^\delta f_j(z))'}{D_q^\delta f_j(z)} - 1 \right) \right] - \xi_j \cos \lambda \right\} \right| d\theta \quad (2.2)$$

Since $f_j \in R_k^\lambda(\xi_j, b, \delta, q)$, $j \in \{1, 2, \dots, m\}$, we have

$$\int_0^{2\pi} \left| \operatorname{Re} \left\{ e^{i\lambda} \left[1 + \frac{1}{b} \left(\frac{z (D_q^\delta f_j(z))'}{D_q^\delta f_j(z)} - 1 \right) \right] - \xi_j \cos \lambda \right\} \right| d\theta \leq (1 - \xi_j) k\pi \cos \lambda, \quad \text{for} \\ (1 \leq j \leq m) \quad (2.3)$$

Applying (2.4) in (2.3), we conclude

$$\int_0^{2\pi} \left| \operatorname{Re} \left\{ e^{i\lambda} \left(1 + \frac{1}{b} \frac{z (D_q^\delta F(z))''}{(D_q^\delta F(z))'} \right) - \gamma \cos \lambda \right\} \right| d\theta \leq k\pi \cos \lambda \sum_{j=1}^m \alpha_j (1 - \xi_j)$$

Subsequently, $F \in V_k^\lambda(\gamma, b, \delta, q)$ with γ is given by (2.1).

By setting $\lambda = 0$, $\delta = q - 1$ in Theorem 2.1, we obtain the following result.

Corollary 2.1 Let $f_j \in R_k(\xi_j, b)$ for $j \in \{1, 2, \dots, m\}$ with $(0 \leq \xi_j < 1)$, $b \in \mathbb{C} - \{0\}$. Also let $\alpha_j > 0$ ($j \in \{1, 2, \dots, m\}$). If

$$0 \leq \\ 1 + \\ \sum_{j=1}^m \alpha_j (\xi_j - 1) < 1, \\ (2.4)$$

then the integral operator F_m defined by (1.5) is in the class $V_k(\gamma, b)$ with γ defined as (2.1).

Theorem 2.2 Let $f_j \in V_k^\lambda(\xi_j, b, \delta, q)$ for $j \in \{1, 2, \dots, m\}$ with $(0 \leq \xi_j < 1)$, $b \in \mathbb{C} - \{0\}$. Also let λ be real with $|\lambda| < \pi/2$, $\alpha_j > 0$, $j \in \{1, 2, \dots, m\}$. If the condition (2.4) satisfied, then the integral operator H defined by (1.8) is in the class $V_k^\lambda(\xi_j, b, \delta, q)$ with γ is defined by (2.1).



Proof. By (1.8), we get

$$(D_q^\delta H(z))' = \left[(D_q^\delta f_1(z))' \right]^{\alpha_1} \dots \left[(D_q^\delta f_m(z))' \right]^{\alpha_m}$$

This equality implies that

$$(D_q^\delta H(z))'' = \sum_{j=1}^m \alpha_j \left[(D_q^\delta f_j(z))' \right]^{\alpha_j} \frac{(D_q^\delta f_j(z))''}{(D_q^\delta f_j(z))'} \times \prod_{r=1, r \neq j}^m \left[(D_q^\delta f_r(z))' \right]^{\alpha_r}.$$

By differentiating the last equality and multiplying by z/b , we have

$$\begin{aligned} \frac{1}{b} \frac{z (D_q^\delta H(z))''}{(D_q^\delta H(z))'} &= \sum_{j=1}^m \alpha_j \frac{1}{b} \frac{z (D_q^\delta f_j(z))''}{(D_q^\delta f_j(z))'} \\ &= \sum_{j=1}^m \alpha_j \left(1 + \frac{1}{b} \frac{z (D_q^\delta f_j(z))''}{(D_q^\delta f_j(z))'} \right) - \sum_{j=1}^m \alpha_j, \end{aligned}$$

or equivalently

$$e^{i\lambda} \left(1 + \frac{1}{b} \frac{z (D_q^\delta H(z))''}{(D_q^\delta H(z))'} \right) = (1 - \sum_{j=1}^{\infty} \alpha_j) e^{i\lambda} + \sum_{j=1}^{\infty} \alpha_j e^{i\lambda} \left(1 + \frac{1}{b} \frac{z (D_q^\delta f_j(z))''}{(D_q^\delta f_j(z))'} \right).$$

Subsequently and adding $\cos \lambda \sum_{j=1}^{\infty} \alpha_j \xi_j$ on the left hand side and taking the real part, we obtain

$$\operatorname{Re} \left\{ e^{i\lambda} \left(1 + \frac{1}{b} \frac{z (D_q^\delta H(z))''}{(D_q^\delta H(z))'} \right) - \gamma \cos \lambda \right\} = \sum_{j=1}^{\infty} \alpha_j \operatorname{Re} \left\{ e^{i\lambda} \left(1 + \frac{1}{b} \frac{z (D_q^\delta f_j(z))''}{(D_q^\delta f_j(z))'} \right) - \xi_j \cos \lambda \right\},$$

where γ is given by (2.1). Integrating the last equation, we have

$$\begin{aligned} &\int_0^{2\pi} \left| \operatorname{Re} \left\{ e^{i\lambda} \left(1 + \frac{1}{b} \frac{z (D_q^\delta H(z))''}{(D_q^\delta H(z))'} \right) - \gamma \cos \lambda \right\} \right| d\theta \\ &\leq \sum_{j=1}^{\infty} \alpha_j \int_0^{2\pi} \left| \operatorname{Re} \left\{ e^{i\lambda} \left(1 + \frac{1}{b} \frac{z (D_q^\delta f_j(z))''}{(D_q^\delta f_j(z))'} \right) - \xi_j \cos \lambda \right\} \right| d\theta \end{aligned} \quad (2.5)$$



Since $f_j \in R_k^\lambda(\xi_j, b, \delta, q)$, $j \in \{1, 2, \dots, m\}$, we have

$$\int_0^{2\pi} \left| \operatorname{Re} \left\{ e^{i\lambda} \left(1 + \frac{1}{b} \frac{z(D_q^\delta f_j(z))''}{(D_q^\delta f_j(z))'} \right) - \xi_j \cos \lambda \right\} \right| d\theta \leq (1 - \xi_j) k\pi \cos \lambda, \quad \text{for } (1 \leq j \leq m). \quad (2.6)$$

Applying (2.6) in (2.5), we conclude

$$\int_0^{2\pi} \left| \operatorname{Re} \left\{ e^{i\lambda} \left(1 + \frac{1}{b} \frac{z(D_q^\delta H(z))''}{(D_q^\delta H(z))'} \right) - \gamma \cos \lambda \right\} \right| d\theta \leq k\pi \cos \lambda \sum_{j=1}^m \alpha_j (1 - \xi_j)$$

Subsequently, $H \in V_k^\lambda(\gamma, b, \delta, q)$ with γ is given by (2.1).

By setting $\gamma = 0$, $\delta = q - 1$ in Theorem 2.2, we obtain the following result.

Corollary 2.2 Let $f_j \in V_k(\xi_j, b)$ for $j \in \{1, 2, \dots, m\}$ with $(0 \leq \xi_j \leq 1)$, $b \in \mathbb{C} - \{0\}$. Also let $\alpha_j > 0$ ($j \in \{1, 2, \dots, m\}$). If

$$0 \leq 1 + \sum_{j=1}^m \alpha_j (\xi_j - 1) < 1,$$

then the integral operator H defined (1.8) is in the class $V_k(\gamma, b)$ with

$$\gamma = 1 + \sum_{j=1}^m \alpha_j (\xi_j - 1)$$

Remark 2.1 In Corollary 2.2, setting

i) $\xi_1 = \xi_2 = \dots = \xi_m = \xi$, we have [[10], Theorem 2.5].

ii) $k=2$, we have [[21], Theorem 3].

Remark 2.2 other work related to q -hypergeometric function and analytic functions can be found in [22], [23], [24], [25], [26]



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الفهرس

الصفحة	اسم الباحث	عنوان البحث	ر.ت
25-3	زهرة المهدي أبوراس فاطمة أحمد قناو	التسرب الدراسي لدي طلاب الجامعات	1
43-26	علي فرج حامد فاطمة جبريل القايد	استعمالات الأرض الزراعية في منطقة سوق الخميس	2
57-44	ابتسام عبد السلام كشيبي	تأثير صناعة الإسمنت على البيئة مصنع إسمنت لبدة نموذجاً دراسة في الجغرافية الصناعي	3
84-58	عطية صالح علي الربيعي خالد رمضان الجربوع منصور علي سالم خليفة	مفهوم الشعر عند نقاد القرن الرابع الهجري	4
106-85	فتحية علي جعفر أمنة محمد العكاشي ربيعة عثمان عبد الجليل	جودة الحياة لدى طلبة كلية التربية بالخميس	5
128-107	Ebtisam Ali Haribash A.A.H. Abd EL-Mwla	An Active-Set Line-Search Algorithm for Solving Multi-Objective Transportation Problem	6
140-129	مفتاح سالم ثبوت	آليات بناء النص عند بدر شاكر السياب قراءة في قصيدة تموز جيكور	7
155-141	مفتاح ميلاد الهديف جمعة عبد الحميد شنيب	الجرائم الالكترونية	8
176-156	Suad H. Abu-Janah	On the fine spectrum of the generalized difference operator over the Hahn sequence space $B(r, s)_h$	9
201-177	فوزية محمد الحوات سالمة محمد ضو	دراسة تأثير التضاد الكيميائي Allelopathy لمستخلصات بعض النباتات الطبية على نسبة الانبات ونمو نبات القمح Triticum aestivum L.	10
219-202	سليمة محمد خضر	الأعداد الضبابية	11
240-220	S. M. Amsheri N. A. Aboutfeerah	On a certain class of P -valent functions with negative coefficients	12
241-253	Abdul Hamid Alashhab	L'écriture de la violence dans la littérature africaine et plus précisément dans le théâtre Ivoirien Mhoi-Ceul comédie en 5 tableaux de Bernard B. Dadié	13
254-265	Shibani K. A. Zaggout F. N	Electronic Specific Heat of Multi Levels Superconductors Based on the BCS Theory	14



266-301	خالد رمضان محمد الجربوع عطية صالح علي الربيعي	أعراض الشعر المستجدة في العصر العباسي	15
302-314	M. J. Saad, N. Kumaresan Kuru Ratnavelu	Oscillation Criterion for Second Order Nonlinear Differential Equations	16
315-336	صالح عبد السلام الكيلاني ساره مفتاح الزني فدوى خليل سالم	القيم الجمالية لفن الفسيفساء عند العرب	17
337-358	عبدالمعظم امحمد سالم	مفهوم السلطة عند المعتزلة وإخوان الصفاء	18
359-377	أسماء حامد عبدالحفيظ اعليجه	مستوى الوعي البيئي ودور بعض القيم الاجتماعية في رفعه لدى عينة من طلاب كلية الآداب الواقعة داخل نطاق مدينة الخمس.	19
378-399	بنور ميلاد عمر العماري	المؤسسات التعليمية ودورها في الوقاية من الانحراف والجريمة	20
400-405	Mohammed Ebraheem Attaweel Abdulah Matug Lahwal	Application of Sawi Transform for Solving Systems of Volterra Integral Equations and Systems of Volterra Integro-differential Equations	21
406-434	Eman Fathullah Abusteen	The perspectives of Second Year Students At Faculty of Education in EL-Mergib University towards Implementing of Communicative Approach to overcome the Most Common Challenges In Learning Speaking Skill	22
435-446	Huda Aldweby Amal El-Aloul	Sufficient Conditions of Bounded Radius Rotations for Two Integral Operators Defined by q-Analogue of Ruscheweyh Operator	23
447-485	سعاد مفتاح أحمد مرجان	مستوى الوعي بمخاطر التلوث البيئي لدى معلمي المرحلة الثانوية بمدينة الخمس	24
486-494	Hisham Zawam Rashdi Mohammed E. Attaweel	A New Application of Sawi Transform for Solving Ordinary differential equations with Variable Coefficients	25
495-500	محمد على أبو النور فرج مصطفى الهدار بشير على الطيب	استخدام التحليل الإحصائي لدراسة العلاقة بين أنظمة الري وكمية المياه المستهلكة بمنطقة سوق الخميس - الخمس	26
501-511	نرجس ابراهيم محمد شنيب	التقييم المنهجي للمواد الرياضية و الاحصائية نسبة الى المواد التخصصية لعلوم الحاسوب	27
512-536	بشري محمد الهيلي حنان سعيد العوراني عفاف محمد بالحاج	طرق التربية الحديثة للأطفال	28
537-548	ضو محمد عبد الهادي فاروق مصطفى ايور اوي زهرة صبحي سعيد نجاح عمران المهدي	دراسة للحد من التلوث الكهرومغناطيسي باستخدام مركب ثاني أكسيد الحديد مع بوليمر حمض الاكتيك	29



549-563	Ali ahmed baraka Abobaker m albaboh Abdussalam a alashhab	Cloud Computing Prototype for Libya Higher Education Institutions: Concept, Benefits and Challenges	30
564-568	Muftah B. Eldeeb	Euphemism in Arabic Language: The case with Death Expressions	31
569-584	Omar Ismail Elhasadi Mohammed Saleh Alsayd Elhadi A. A. Maree	Conjugate Newton's Method for a Polynomial of degree $m+1$	32
585-608	آمنة سالم عبد القادر قدرو آلاء عبدالسلام محمد سويسي ليلى علي محمد الجاعوك	الصحة النفسية وعلاقتها بتقدير الذات لدى عينة من طلبة كلية الآداب والعلوم / مسلاته	33
609-625	نجاه سالم عبد الله زريق	المساندة الاجتماعية لدى عينة من المعلمات بمدينة قصر الأخبار وعلاقتها ببعض المتغيرات الديموغرافية "دراسة ميدانية"	34
626-640	محمد سالم ميلاد العابر	"أي" بين الاسمية والفعلية عاملة ومعمولة	35
641-659	إبراهيم فرج الحويج	التمييز في القرآن الكريم سورة الكهف أنموذجا	36
660-682	عبد السلام ميلاد المركز رجعة سعيد الجنقاوي	الموارد الطبيعية و البشرية السياحية بمدينة طرابلس (بليبيا)	37
683-693	Ibrahim A. Saleh Abdelnaser S. Saleh Youssif S M Elzawiei Farag Gait Boukhrais	Influence of Hydrogen content on structural and optical properties of doped nano-a-Si:H/a-Ge: H multilayers used in solar cells	38
694-720	فرج رمضان مفتاح الشبيلي	أجوبة الشيخ علي بن أبي بكر الحضيري (ت: 1061 هـ - 1650 م)	39
721-736	علي خليفة محمد أجولي	مفهوم الهوية عند محمد أركون	40
737-742	Mahmoud Ahmed Shaktour	Current –mode Kerwin, Huelsman and Newcomb (KHN) By using CDTA	41
743-772	Salem Msauad Adrugi Tareg Abdusalam Elawaj Milad Mohamed Alhwat	University Students' Attitudes towards Blended Learning in Libya: Empirical Study	42
773-783	Alhusein M. Ezarzah Aisha S. M. Amer Adel D. El werfalyi Khalil Salem Abulsba Mufidah Alarabi Zagloom	Integrated Protected Areas	43
784-793	عبد الرحمن المهدي ابومنجل	المظاهرات بين المانعين والمجوزين	44
794-817	رضا القذافي بشير الاسمر	ترجيحات الامام الباجي من خلال كتابه المنتقى " من باب العنافة والولاء الى كتاب الجامع "	45



مجلة التربوي
Journal of Educational
ISSN: 2011- 421X
Arcif Q3

معامل التأثير العربي 1.5
العدد 20

818-829	Fadela M. Elzalet Sami A. S. Noba omar M. A. kaboukah	IDENTIFICATION THE OPTIMUM PRODUCTION PROCESS OF THE HYDROGEN GAS	46
830	الفهرس		