Maximum Area Aggregation Approach For Cumulant-Based Probabilistic Optimal Power Flow studies

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ABSTRACT

The paper introduces a Maximum Area Aggregation (MAA) approach for Cumulant-Based Probabilistic Optimal Power Flow (P-OPF) studies. The Maximum Area Aggregation (MAA) approach relies on the Cumulant Method (CM) to produce Probability Density Functions (PDFs) in the limited and the original cases, and then combines these PDFs to generate the final PDF for all system variables. The probabilities that system variables reach their limits are computed and the maximum probability is extracted and used to find the final PDF by aggregating the PDFs (the original PDFs and the limited ones). The proposed approach is verified against Monte-Carlo Simulation (MCS) consisting of 10,000 samples and compared with the original Cumulant Method (CM). The results of MAA approach demonstrate significant improvements when compared with traditional CM results.

Keywords: Maximum Area Aggregation (MAA), Monte Carlo Simulation (MCS), Optimal Power Flow, Probabilistic Optimal Power Flow (P-OPF), Probability Density Functions (PDFs).

1 Introduction

Power systems are stochastic in nature, this fact made most publications introduce computational methods for solving power flow problems using only deterministic Optimal Power Flow (OPF) approaches [1]–[5]. However, in recent years, Probabilistic Optimal Power Flow (P–OPF) problems have been developed, which address random quantities such as bus loading and generator bids, (for example [6]–[13]). Probabilistic Optimal Power Flow (P–OPF) seeks a distribution for system variables and represents this distribution using Probability Density Function (PDF) [14]–[16]; that is, each individual point on the PDF represents an optimal solution for a certain randomly generated input.

Probabilistic techniques have been used to account for random quantities within power systems since the early seventies [2]. There are many papers published regarding the introduction of random quantities, such as power demand, into the ordinary Power Flow (PF) problems [17]–[19].

Monte Carlo Simulation (MCS) technique is an easy way to account for uncertainty in power systems [17], [20]–[22]. Although this technique provides accurate results, and it is easy to implement, it involves a large number of trials, which makes it computationally expensive and time consuming. The computational expense of MCS has led to the foundation of alternative methods such as the Cumulant Method [9], [10], [23]–[29]. The Cumulant Method has been shown to be well suited for traditional OPF type problems. In [11], the authors introduce an adaptation of a Two-Point Estimate Method (2PEM), which was proposed in [30], to a market based P–OPF. Recently, a P-OPF was studied including wind generator bids as in [31]. In [12], a comparison of the two-point estimate method (2PEM) and the CM to find PDFs of Locational Marginal Prices (LMP) in a deregulated electricity market is introduced. When applied to a problem based on electricity markets (where generator bids are considered as the input uncertainty), the CM method tends to produce poor results as the CM cannot capture discrete changes in the merit order. In this paper, a new solution approach, called the Maximum Area Aggregation (MAA), to a market-based problem is proposed based on the CM. The Maximum Area Aggregation (MAA) approach relies on the CM to produce Probability Density Functions (PDFs) in the limited and the original cases, and then combines these PDFs to generate the final PDF for all system variables.

2 Cumulant Method For P-OPF.

This section provides a brief background on the adaptation of the Cumulant Method to a Market-Based P–OPF to highlight the limitation of the standard CM when applied to such problems. Further details on the fundamentals of the Cumulant Method can be found in [9], [10], [26]–[29], [32]. The Cumulant Method (CM) is based on linearly mapping cumulants of known random variables (bus loading) into the cumulants of unknown random variables (active and reactive power generation, bus voltage magnitudes, and phase angles) [9], [14].

2.1 Mathematical Formulation of CM

Given a random variable x with a Probability Density Function (PDF) $f_X(x)$, the mean μ_x is the expected value of x and is stated as [14]:

$$\mu_x = E[x] = \int_{-\infty}^{\infty} x f_X(x) dx \tag{1}$$

Cumulants are a statistical measure of a random variable and are calculated based on the cumulant generating function $\Psi_X(s)$. The cumulant generating function $\Psi_X(s)$ is defined as the natural logarithm of the moment generating function and is written as:

$$\Psi_X(s) = \ln(\Phi_x(s)) \tag{2}$$

Where the moment generating function $\Phi_x(s)$ is defined as:

$$\Phi_x(s) = E(e^{sx}) \tag{3}$$

The n^{th} order moment is computed by taking the n^{th} derivative of (3) with respect to s and evaluating at s = 0.

Therefore, to generate the n^{th} order cumulant λ_n for the random variable x, the n^{th} derivative of equation (2) with respect to s is taken and evaluated at s = 0.

Since any given system includes many random inputs, the combination of two or more random variables is essential [32]. The linear combination of m known and independent random variables $(x_1, x_2, \dots \dots \dots . x_m)$ is given by:

$$z = a_1 x_1 + a_2 x_2 + \cdots + a_m x_m \tag{4}$$

where z is a new random variable and a_1, a_2, ..., a_m are the mapping coefficients.

In market-based systems, prices are no longer modelled as deterministic variables; rather, prices are modelled as random variables. As a consequence, Locational Marginal Prices (LMP) change in response to changes in bids.

A simple deterministic OPF problem formulation based on a bidding based model [33] is given as:

$$\begin{aligned} min \quad & C^T P_{gen} \\ s.t \quad & \mathcal{F}(X) = 0 \\ & X \geq X_{min} \\ & X \leq X_{max} \end{aligned} \tag{5}$$

where $\boldsymbol{C}^T P_{gen}$ is the objective cost function, $\boldsymbol{C}$ is a vector of generator bids, P_{gen} is a vector of the active power generation, $\mathcal{F}(X)$ is a vector of AC power flow equations, X represents the unknown variables (active and reactive power generation, voltage magnitudes and phase angles), and X_{min}, X_{max} are the lower and upper limits.

Equation (5) represents the main market-based P–OPF problem (i.e. minimizing the cost of power generation). The formulation of such problems and the CM solution technique follows a similar pattern to that presented in [9], [10], [32]. This technique is based on incorporating bus loading and generator bids as random variables into the OPF problem. The first-order KKT optimality conditions of (5) are found and written as:

$$F(\boldsymbol{Y}, \boldsymbol{L}, \boldsymbol{C}) = 0 \tag{6}$$

where $F(\cdot)$ is the set of equations defining the first-order KKT conditions, $\boldsymbol{Y}$ is the vector of conventional problem variables including slack variables and Lagrange multipliers, $\boldsymbol{L}$ is the vector of bus loading, and $\boldsymbol{C}$ is the vector of generator bids.

Lagrange Multipliers associated with AC power-flow equations as equality constraint, γ_1, are of special interest. Those Lagrange Multipliers are directly related to the costs C [33]. Moreover, γ_1 represents a vector of spot prices in the system, known as Locational Marginal Prices (LMP).

In order to find the cumulants of the conventional problem variables, $\boldsymbol{Y}$, a linear relationship between $\boldsymbol{Y}, \boldsymbol{L}$, and $\boldsymbol{C}$ is developed by taking the full derivative of (6) as follows:

$$\mathcal{H}\Delta\boldsymbol{Y} + \Delta\hat{\boldsymbol{L}} + \Delta\hat{\boldsymbol{C}} = 0 \tag{7}$$

where

$$\Delta\hat{\boldsymbol{L}} = [0\ 0\ 0\ 0\ \boldsymbol{\Delta L}\ 0\ 0\ 0\ 0\ 0]^T \tag{8a}$$

$$\Delta\hat{\boldsymbol{C}} = [0\ 0\ 0\ 0\ \ 0\ 0\ 0\ 0\ 0\ \boldsymbol{\Delta C}]^T \tag{8b}$$

and $\boldsymbol{\Delta L}$ and $\boldsymbol{\Delta C}$ are vectors of the changes in the input random variables (bus loading and bid prices). The expression (7) can be rearranged into the following form:

$$\Delta\mathrm{Y} = -\mathcal{H}^{-1}\Delta\hat{\mathrm{L}} - \mathcal{H}^{-1}\Delta\hat{\mathrm{C}} \tag{9}$$

Therefore, for areas around the mean solution, changes in system variables Y are linearly related to changes in bus loading L and generator bids C. The negative inverse of the Hessian of the Lagrangian matrix, $-\mathcal{H}^{-1}$, is used to map cumulants of the input random variables into cumulants of system outcomes.

Once the cumulants for the conventional problem variables are found utilizing (4) and (9), PDFs for these variables are then rebuilt using the Gram-Charlier series theory [9].

2.2 Comparison Methods

Comparison between the proposed approaches' results and the MCS is done through the use of the Normalized Sum of Square Error (NSSE) and the Averaged NSSE (ANSSE) [34]. In order to calculate NSSE, the sum of square error (SSE) is found using:

$$SSE = \sum_{i=1}^{n}(f_r(i) - f_c(i))^2 \tag{10}$$

where $f_r(i)$ is the reference value at point *i* (based on a 10,000 sample MSC), $f_c(i)$ is the computed value at *i*, and *n* is the total points taken for comparison. The NSSE is then computed as:

$$NSSE = \frac{\sum_{i=1}^{n}(f_r(i)-f_c(i))^2}{\sum_{i=1}^{n}(f_r(i))^2} \tag{11}$$

Finally, the ANSSE is computed using:

$$ANSSE = \frac{\sum_{i=1}^{m} NSSE_i}{m} \tag{12}$$

where *m* is equal to the number of generator buses (gb) for active and reactive power error calculation, *m* is equal to the number of buses (nb) for voltage magnitude error calculation, and *m* is equal to the number of non-slack buses (nsb) for phase angle error calculation.

3 Maximum Area Aggregation (MAA) Approach.

The original Cumulant Method presented in [9], [10], [32] does not take active limits, away from the mean solution, into account while solving the P-OPF problem. To account for limits becoming active away from the mean using the CM, the P-OPF problem is solved again with the variable being fixed at its active limit. The PDFs obtained with the variable being fixed at its active limit show the behaviour of the system and provide information relevant to a case when the limit is active. The original PDFs, together with the PDFs obtained with the variable being fixed at its active limit, are then aggregated to generate the final corrected PDFs. Probability Density Functions generation and aggregation is summarized in the following steps:

1) Probability density functions of all unknown variables are found using the standard CM described in [26] and [32].

$$\mathrm{F}_{\mathrm{X_o}}(\mathrm{X_o}) = \begin{bmatrix} \mathrm{F}_{\mathrm{V_o}}(\mathrm{V_o}) \\ \mathrm{F}_{\delta_o}(\delta_o) \\ \mathrm{F}_{\mathrm{P_o}}(\mathrm{P_o}) \\ \mathrm{F}_{\mathrm{Q_o}}(\mathrm{Q_o}) \end{bmatrix} \tag{13}$$

   Where $F_{V_o}(V_o) \in \mathcal{R}^{nb\times 1}$, $F_{\delta_o}(\delta_o) \in \mathcal{R}^{(nb-1)\times 1}$, $F_{P_o}(P_o) \in \mathcal{R}^{ngb\times 1}$, $F_{Q_o}(Q_o) \in \mathcal{R}^{ngb\times 1}$ are the probability density functions of voltage magnitudes, phase angles, active power generation, and reactive power generation respectively.

2) Active limits in the system are found by computing the areas probabilities [14, 15] that the variables values exceed the imposed limits. This is done by integrating the original PDFs, $F_{X_o}(X_o)$, past the upper limits as follows [14]:

$$\breve{\alpha}_i = \int_{X_{max_i}}^{\infty} x_i f_{X_i}(x_i)dx \quad (14)$$

where $\breve{\alpha}_i$ is the area under the PDF that extends past the limit X_{max_i} for the random variable X_i. It is noteworthy that if the lower limit is active, the area $\breve{\alpha}_i$ (14) becomes:

$$\breve{\alpha}_i = \int_{-\infty}^{X_{min_i}} x_i f_{X_i}(x_i)dx \quad (15)$$

where $\breve{\alpha}_i$ is the area under the PDF that extends past the limit X_{max_i} for the random variable x_i.

The value of $\breve{\alpha}_i$ indicates the ratio, expressed out of 1, that the variable exceeds its limit; that is, the variable value is greater/smaller than the limit for x_i. For *m* variables, the vector *A* is written as:

$$A = [\breve{\alpha}_1 \quad \breve{\alpha}_2 \quad \cdot \quad \cdot \quad \cdot \quad \breve{\alpha}_m]^T \quad (16)$$

For the problems considered in this paper, m equals two times the number of generator buses plus two times the number of buses in the system minus 1, since the angle of the slack bus is assumed known (i.e. the voltage angle reference). Since the upper limit in (14) is ∞ and the lower limit in (15) is -∞, none of the elements in A will equal zero. Hence, the value 0.1 was chosen as the largest negligible area; that is, the areas that are smaller than 0.1 will not be considered, since they have no significant effect. Although this value was chosen based on experience, it was found that the sensitivity of the results to this choice is low (for example, setting the threshold of 0.11 or 0.09 has negligible impact on the final results).

3) Since the areas presented in the set *A* are equal to the probabilities that the variables reach their limits, the variable corresponding to the largest element in *A* is most likely to reach its limit first. Hence, the limit corresponding to the largest element in *A* is assumed to be the cause of the distortion. Accordingly, for the largest element in *A*, $\breve{\alpha}_{max}$ where $\breve{\alpha}_{max} \geq 0.1$, the CM is then applied again to obtain new PDFs, $F_{X_{\breve{\alpha}_{max}}}(X_{\breve{\alpha}_{max}})$, corresponding to the case when the limit is active; that is, the P-OPF problem is solved with the variable corresponding to $\breve{\alpha}_{max}$ being fixed at its active limit. Solving the P-OPF problem with the variable corresponding to $\breve{\alpha}_{max}$ being fixed at its active limit results in new PDFs associated with the limited case. Using this approach, the following PDFs are found:

$$F_{X_{\breve{\alpha}_{max}}}\left(X_{\breve{\alpha}_{max}}\right) = \begin{bmatrix} F_{V_{\breve{\alpha}_{max}}}\left(V_{\breve{\alpha}_{max}}\right) \\ F_{\delta_{\breve{\alpha}_{max}}}\left(\delta_{\breve{\alpha}_{max}}\right) \\ F_{P_{\breve{\alpha}_{max}}}\left(P_{\breve{\alpha}_{max}}\right) \\ F_{Q_{\breve{\alpha}_{max}}}\left(Q_{\breve{\alpha}_{max}}\right) \end{bmatrix} \tag{17}$$

4) In this step, the PDFs obtained in Step 1, (13), and in Step 3, (17), are aggregated together to form the final corrected PDFs, $F_{X_c}(X_c)$. Since the scaler $\breve{\alpha}_{max}$ presents the probability that the system is limited, the probability that the system is not limited is equal to the scalar (1-$\breve{\alpha}_{max}$). Hence, the aggregation to find the final corrected PDFs is done by adding (1-$\breve{\alpha}_{max}$) , $F_{X_o}(X_o)$ to $\breve{\alpha}_{max}F_{X_{\breve{\alpha}_{max}}}\left(X_{\breve{\alpha}_{max}}\right)$ as follows:

$$F_{X_c}(X_c) = (1-\breve{\alpha}_{max})\ F_{X_o}(X_o) + \breve{\alpha}_{max}F_{X_{\breve{\alpha}_{max}}}\left(X_{\breve{\alpha}_{max}}\right) \tag{18}$$

Note that equation (18) produces new PDFs which are combination of the original PDFs and the limited ones. This aggregation, (18), is applied to all unknown variables, except for the variable that corresponds to $\breve{\alpha}_{max}$ in the set A.

- For any variable that reaches a limit, the integration (14) is utilized to correct the original PDFs into the final corrected PDFs. The use of integration to correct the limited PDFs is summarized in the following steps:
  a) Compute the area under the original PDF that extends past the limit. For the case where the distribution extends past an upper limit, (14) can be used to compute the area to the right of the limit. For the case where the distribution extends past a lower limit, (15) can be used to compute the area to the left of the limit.
  b) The excess area is compressed to the limit imposed. The overall area under the PDF is still unity but the result is no longer a smooth function. However, the MCS results are also limited by the constraint in the problem and exhibit non-smooth behaviour as well.
  c) In order to compare the P-OPF result with the MCS histogram, the probability associated with the limited value is divided by the bin width of the histogram so that the amplitudes are comparable. Once this is completed, the final bin of the histogram can be directly compared with the PDF computed by the P-OPF method and corrected through the use of integration.

4 Numerical Results

Numerical results presented in this paper are based on applying the CM using the MAA approach to the economic P-OPF problem to the IEEE 30-bus system [35]. A MATLAB program is used for simulation.

It is noteworthy that the CM results and results of the CM using the MAA approach are exactly the same if the original IEEE 30-bus data file is used; that is, without modifying the lower/upper limits, no limits are encountered. Hence, the upper limit of the active power generation at bus 5 was reduced from 1 p.u to 0.3 p.u to force the upper limit at this particular bus to be active. With this modification to the upper limit of the active power generation at bus 5, there is significant distortion to the results for other system variables.

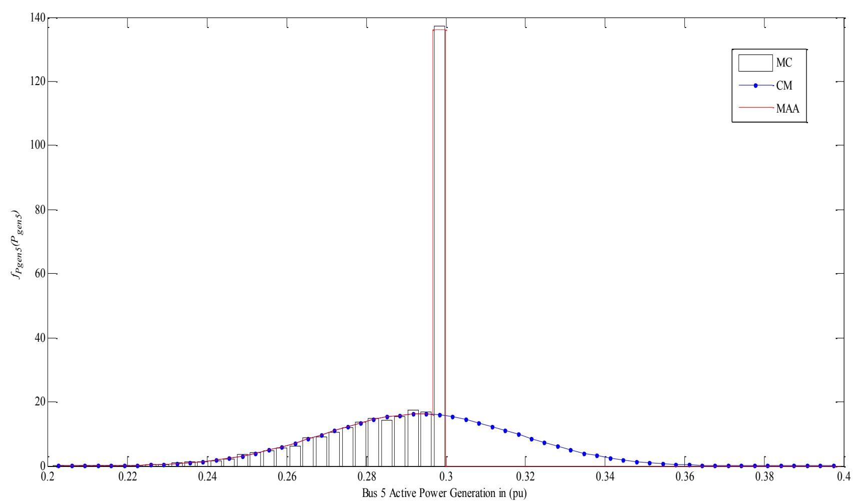

Figure 1: *IEEE 30 Bus Test System, Probability Density Function of Active Power Generation at bus 5, $f_{P_{gen5}}(P_{gen5})$, Results of the CM Using Maximum Area Aggregation*

Figure 1 depicts the final corrected PDF of active power generation at bus 5, when the upper limit is active. The dotted line represents the final corrected PDF, $F_{X_c}(X_c)$, the solid line represents the original uncorrected PDF , $F_{X_o}(X_o)$, and the histogram represents MCS. From Figure 1, results of the CM using the MAA approach are much closer to MCS than the original CM results.

Figure 2 depicts the final corrected PDF of active power generation at bus 8 when the upper and lower limits are not active. It is noteworthy that the MAA approach accurately estimates

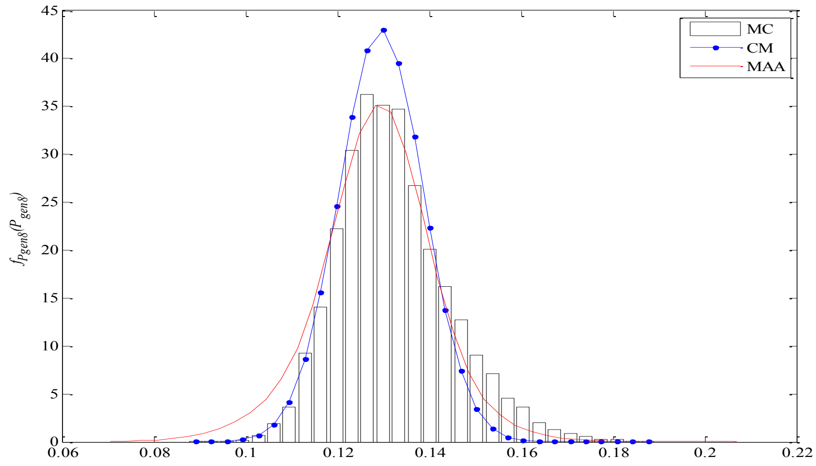

Figure 2: *IEEE 30 Bus Test System, Probability Density Function of Active Power Generation at bus 8, $f_{P_{gen8}}(P_{gen})$, Results of the CM Using Maximum Area Aggregation*

PDFs of system variables that have active limits, yet fails to accurately estimate PDFs of other system variables. Variables that have active limits are corrected using the integration, (14). Hence, a significant improvement is noticed, see Figure 1. The other variables, however, are corrected using (18).

The way the formula (18) combines the PDFs causes insignificant improvement in the results; that is, (18) combines both the limited and the original curves in their entirety. This approach is likely to produce satisfactory and accurate results compared with the CM against MCSs.

Table 1: *IEEE 30 Bus Test System, ANSSE of Optimal Power Flow Variables for the CM and the MAA*

Power System Variables	ANSSE (CM)	ANSSE (MAA)
Active Power	0.1964	0.0240
Reactive Power	0.0031	0.0029
Voltage Magnitude	0.0086	0.0066
Phase Angle	0.0296	0.0180

Average error results for the active and reactive power generation, voltage magnitudes, and phase angles, for the CM and the MAA approach. It is noteworthy that the ANSSE of the active power generation improved from 19:64% to 2:40% and for the phase angles from 2:96% to 1:8% for the CM and the MAA respectively. The ANSSE of reactive power generation and voltage magnitudes, for the CM and MAA, are also improved but not as well though.

5 Conclusion

A Maximum Area Aggregation (MAA) approach for Cumulant-Based Probabilistic Optimal Power Flow (P-OPF) studies is introduced. The new approach relies on the Cumulant Method (CM) to produce Probability Density Functions (PDFs) in the limited and the original cases, and then combines these PDFs to generate the final PDF for all system variables. The probabilities that system variables reach their limits are computed and the maximum probability is extracted and used to find the final PDF by aggregating the PDFs (the original PDFs and the limited ones). A MATLAB program is written and used to verify the proposed approach against Monte-Carlo Simulation (MCS) consisting of 10,000 samples and compared with the original Cumulant Method (CM). The Averaged NSSE (ANSSE) is computed and used to present the improvement. The results of MAA approach demonstrate significant improvements when compared with traditional CM results.

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