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كلية التربية / الخمس

جامعة المرقب

العدد الثاني والعشرون

يناير 2023م

- المجلة ترحب بما يرد عليها من أبحاث وعلى استعداد لنشرها بعد التحكيم .
- المجلة تحترم كل الاحترام آراء المحكمين وتعمل بمقتضاها .
- كافة الآراء والأفكار المنشورة تعبر عن آراء أصحابها ولا تتحمل المجلة تبعاتها .
- يتحمل الباحث مسؤولية الأمانة العلمية وهو المسؤول عما ينشر له .
- البحوث المقدمة للنشر لا ترد لأصحابها نشرت أو لم تنشر .

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ضوابط النشر:

يشترط في البحوث العلمية المقدمة للنشر أن يراعى فيها ما يأتي :

-أصول البحث العلمي وقواعده .

-ألا تكون المادة العلمية قد سبق نشرها أو كانت جزءا من رسالة علمية .

-يرفق بالبحث تزكية لغوية وفق أنموذج معد .

-تعدل البحوث المقبولة وتصحح وفق ما يراه المحكمون .

-التزام الباحث بالضوابط التي وضعتها المجلة من عدد الصفحات ، ونوع الخط ورقمه ، والفترات الزمنية الممنوحة للتعديل ، وما يستجد من ضوابط تضعها المجلة مستقبلا .

تنبيهات :

-للمجلة الحق في تعديل البحث أو طلب تعديله أو رفضه .

-يخضع البحث في النشر لأولويات المجلة وسياستها .

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Oneness and existence of the solution to the problem of boundary values for a set of second-order partial differential equations

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كلية طرابلس للعلوم والتقنية

Abstract: Most of the problems of mathematical physics, when solving them, result in solving one or more partial differential equations with imposed initial and boundary conditions. This is known as boundary value problems for differential equations. This paper studies the solution of a set of parabolic and hyperbolic partial differential equations with boundary conditions imposed in different regions of the y o x plane

Keywords: differential equations, boundary value problems.

1. INTRODUCTION:

The research deals with the theory of mixed differential equations, which is one of the theories currently treated by partial differential equations, and although it was first treated by the Italian mathematician Tricomi in the forties of the last century, interest in it did not begin in earnest, until the late seventies of the century The same, when the Dutch mathematician Frankl pointed out their importance in solving some problems related to the movement of gases and liquids and the curvatures of surfaces.

And the main founders of the problems of boundary values of partial differential equations are among them Petsadze [2], Guriev [3], Kirievov [7], Sapoev [4], Alimov and Bulatov [1].

We find boundary-value problems of parabolic differential equations in works [3], [8], and we find boundary-value problems of parabolic-hyperbolic equations in works [3], [4], [7].

As for this research, it is an extension of the research published for [one], [three], [eight], and Salah al-Dinouf, and the oneness theorem has been proven and the existence of a solution to the problem of boundary values for a set of partial differential equations of the second order, as well as finding the functions that achieve these Equations in certain squares and with imposed boundary conditions, and here I also prove that when some conditions are met, the existence of the solution requires its oneness. She also attributed the solution to this boundary problem to the solution of a system of linear integral equations of the second type, Friedholm

2. BASIC DEFINITIONS

- **Definition of Kielder's condition:**

We say: The function $\boldsymbol{a(x,y)}$ satisfies the Kielder condition with the coefficient $\boldsymbol{(a)}$ where $\boldsymbol{0} < a \leq 1$, If a positive constant k is found, the following inequality is satisfied.

$$|\mathbf{a}(x,y_1) - a(x,y_2)| \leq k|y_1 - y_2|^{\alpha}$$

For any two points $(x,y_1), (x,y_2)$ belonging to a square $\boldsymbol{G}$.

- **Define a regular solution**

A solution to a differential equation is called regular if it and its partial derivatives are continuous up to the second order implicit throughout problem $\boldsymbol{G}$.

- **Definition of Friedholm's Linear Equation:**

If the linear integral equation is of the form:

$$v(y) - \lambda \int_a^b k\,(y,\eta) v(\eta) d\eta = p(y) \qquad (A)$$

whereas:
$k(y,\eta)$ is called the nucleus of the integral equation. It is a known function of the independent variables η and y.
$P(y)$ is the right side of the equation and is a known function (the free side).
$v(y)$ is an unknown function that we want to determine.
λ is a scalar mediator.
It is called Friedholm's linear integral equation of the second kind.

- **Transferred Integral Equation:**

If the integral equation is of the form:

$$v_1(y) - \lambda \int_a^b k(\eta, y) v_1(\eta) d\eta = p_1(y) \qquad (B)$$

It is called the integral equation transferred for (A)
Let us note that the nucleus in (B) does not differ from the nucleus in (A)except that y and η they exchange their positions.

- **Friedholm's Hypothesis in Integrative Equations**

Let us distinguish the following two cases: (A) let us have the equation.

a. If λ is not a distinct value for the nucleus $K(Y,\eta)$ then the homogeneous integral equation and its vector have only the zero solution, and equation (A) and its vector have only one solution.
b. If λ is a characteristic value for the nucleus $K(Y,\eta)$, then the homogeneous equation corresponding to (A) has solutions other than the zero solution that form a space with a finite dimension, just as the transposon of this homogeneous equation also has solutions other than the zero solution that form a space of the same dimension.

- **Definition of kernel state**

Let's look at the following series:
$H_1(y,\eta) + \lambda H_2(y,\eta) + \lambda^2 H_3(y,\eta) + \cdots + \lambda^{m-1} H_m(y,\eta) + \cdots.$
This series is a power series in λ and is symbolized by the symbol $R(y,\eta,\lambda)$, and the nucleus is called the state by $H(y,\eta)$, and from it:
$R(y,\eta,\lambda) = H_1(y,\eta) + \lambda H_2(y,\eta) + \lambda^2 H_3(y,\eta) + \cdots + \lambda^{m-1} H_m(y,\eta) + \cdots$
As for the solution form of the Friedholm integral linear equation of the second type using the state nucleus, it is given by the equation:

$$v(y) = P(y) + \lambda \int_a^b R(y,\eta,\lambda) P(\eta) d\eta$$

Among the important formulas achieved by the state kernel is the following formula:

$$R(y,\eta,\lambda) = H_1(y,\eta) + \lambda \int_a^b H_1(y,x) R(x,\eta,\lambda) dx$$

3. Presentation of the issue

Let's look at the following set of equations:

1. $U_{xx} - U_y + a(x,y)U_x + b(x,y)U = 0 \qquad ; (x,y) \in D_1$
2. $U_{xx} - U_{yy} + a_1 U_x + c_1 U = 0 \qquad ; (x,y) \in D_2$
3. $U_{xx} - U_{yy} + a_2 U_x + b_2 U_y + c_2 U = 0 \qquad ; (x,y) \in D_3$

This is assuming that $a(x,y)$ is a known continuous function that is differentiable only once in square $\overline{D}_1$ and Kielder's condition is fulfilled by coefficient α [9]. And where $0 < \alpha \le 1$, and $b(x,y)$ is a known continuous function that fulfills Kielder's condition by coefficient α in square $\overline{D}_1$.
where $\overline{D}_1$ is an open square, limited by the lines A_0A, B_0A_0, BB_0, AB whose equations, respectively:
$y = 0, x = 1, y = 1, x = 0$ (ie D_1) is the square $\{0 < x < 1, 0 < y < 1\}$).
where $A_0(0,1), B_0(1,1), B(1,0), A(0,0)$ and D_2 are a triangle limited by the lines AA_0, AC, A_0C whose equations are, respectively:
$x = 0, x + y = 0, y - x = 1$ and where $C(-\frac{1}{2}, \frac{1}{2})$, It represents an open space.
And D_3 is a triangle limited by lines BB_0, BE, B_0E, whose equations are, in order:
$x = 1, x - y = 1, x - 2 = -y$ **and where** $E(\frac{3}{2}, \frac{1}{2})$, **It is also an open space.**
As for $a_1, a_2, b_1, b_2, c_1, c_2$, They are optional constants.
Let's denote by $\overline{D}$ the D label.

$$D = D_1 \cup AA_0 \cup\cup BB_0 \cup D_2 \cup D_3$$

If we perform in (1) the following transformation:

$$U(x,y) = V(x,y)exp\left\{-\frac{1}{2}\int_0^x a\,(t,y)dt\right\}$$

We get the equation:

$$V_{xx} - V_x + C(x,y)V = 0$$

Where:

$$C(x,y) = -\frac{1}{4}a^2(x,y) - \frac{1}{2}\frac{\partial}{\partial x}a(x,y) + \frac{1}{2}\frac{\partial}{\partial y}\int_0^x a\,(t,y)dt + b(x,y)$$

Likewise, if we perform in (2) the following transformation:

$$U(x,y) = V(x,y)e^{\alpha_1 x + \beta_1 y}$$

We get the equation:

$$V_{xx} - V_{yy} + \lambda_1 V = 0$$

This is assuming that:

$$\lambda_1 = \frac{1}{4}(4c_1^2 - a_1^2 - b_1^2);\ \alpha_1 = \frac{-a_1}{2}\ ;\ \beta_1 = \frac{b_1}{2}$$

Similarly, if in (3) we perform the following transformation:

$$U(x,y) = V(x,y)e^{\alpha_2 x + \beta_2 y}$$

We get the equation:

$$V_{xx} - V_{yy} + \lambda_2 V = 0$$

Where

$$\lambda_2 = \frac{1}{4}(4c_2 - \mathrm{a}_2^2 + b_2^2)\ ;\ \alpha_2 = -\frac{\mathrm{a}_2}{2}\ ; 2 = \frac{b_2}{2}$$

For these reasons, it is sufficient to search for the following set of equations:

4. $U_{xx} - U_y + C(x,y)U = 0 \qquad ; (x,y) \in D$
5. $-U_{xx} - U_{yy} - \lambda_1 U = 0 \qquad ; (x,y) \in D_2$
6. $-U_{xx} + U_{yy} - \lambda_2 U = 0 \qquad ; (x,y) \in D_3$

4. Matter (N)

It is required to search for the regular solution $U(x,y)$ for equations (4), (5) and (6) in square D except for the points of the straight lines AA_0 and BB_0 which fulfills the condition:

$$U(x,y) \in c(\bar{D}_1) \cap [e^1(D_2 \cup AA_0) \cap e^1(D_3 \cup BB_0) \cap e^1(D_1 \cup AA_0 \cup BB_0)]$$

And also for the following boundary conditions:

7. $U|_{A_0c} = \Psi_1(y)\,; \qquad \frac{1}{2} \le y \le 1$

8. $U|_{BE} = \Psi_2(y)\,; \qquad 0 \le y \le \frac{1}{2}$

9. $U|_{y=0} = \varphi(x)\,; \qquad 0 \le x \le 1$

$U(-0,y) = \alpha_1(y)\,U(+0,y) + \gamma_1(y)$

$U_x(-0,y) = \beta_1(y)U_x(+0,y) + \delta_1(y)U(+0,y) + \sigma_1(y)$

10. $U(1+0,y) = \alpha_2(y)U(1-0,y) + \gamma_2(y)$

$U_x(1+0,y) = \beta_2(y)U_x(1-0,y) + \delta_2(y)U(1-0,y) + \sigma_2(y)$

This is assuming that:

$\alpha_1(y), \alpha_2(y), \beta_1(y), \beta_2(y), \gamma_1(y), \gamma_2(y), \sigma_1(y), \sigma_2(y), \delta_1(y), \delta_2(y), \Psi_1(y), \Psi_2(y), -\varphi(x)$

Given and continuous functions. Moreover, the functions

$\alpha_1''(y), \varphi'(x), \Psi_2''(y), \Psi_1''(y), \gamma_1''(y), \gamma_2''(y), \beta_2''(y), \beta_1''(y), \alpha_2''(y)$ ongoing.

Let us introduce the following hypotheses:

11. $$\begin{cases} U(+0,y) = \tau_1^+(y), U_x(+0,y) = v_1^+(y), \\ U(-0,y) = \tau_1^-(y), U_x(-0,y) = v_1^-(y), \\ U(1+0,y) = \tau_2^-(y), U_x(1+0,y) = v_2^-(y) \\ U(1-0,y) = \tau_2^+(y), U_x(1-0,y) = v_2^+(y) \end{cases}$$

As in work [4], on the lines AA_0 and BB_0, we get the two basic dependent relationships between the functions $\bar{\tau}_1(y)$ and $\bar{v}_1(y)$ on the one hand and between the functions $\bar{\tau}_2(y)$ and $\bar{v}_2(y)$ on the other hand, in squares D_2 and D_3, respectively, as follows:

12. $\bar{\tau}_1(y) = \rho_1(y) + \int_y^1 J_0\,[\lambda_1(y-t)]\bar{v}_1(t)dt, \quad 0 < y < 1$

13. $\bar{\tau}_2(y) = \rho_2(y) + \int_0^y J_0\,[\lambda_0(y-t)]\bar{v}_2(t)dt, \quad 0 < y < 1$

Where:

$$\rho_1(y) = 2\Psi_1\left(\frac{y+1}{2}\right) - \Psi_1(1) + \int_y^1 \frac{\partial}{\partial t} J_0\left(\lambda_1\sqrt{(y-1)(y-t)}\right)\left[2\Psi_1\left(\frac{t+1}{2}\right) - \Psi_1(1)\right]dt;$$

$$\rho_2(y) = 2\Psi_2\left(\frac{y}{2}\right) - \Psi_2(0) - \int_0^y \frac{\partial}{\partial t} J_0\left(\lambda_2\sqrt{t(t-y)}\right)\left[2\Psi_2\left(\frac{t}{2}\right) - \Psi_2(0)\right]dt;$$

where : J_0 is a Bessel function of the first type and rank zero.

5. Theorem

If the following conditions are met:

14. $C(x,y) \le 0\,;\,(x,y) \in D_1$

15. $\frac{1}{\alpha_1(0)\beta_1(0)} > 0\,, \frac{d}{dy}\left[\frac{1}{\alpha_1(y)\beta_1(y)}\right] \ge 0\,,\ \frac{\delta_1(y)}{\beta_1(y)} \le 0$

16. $\frac{1}{\alpha_2(1)\beta_2(1)} > 0\,, \frac{d}{dy}\left[\frac{1}{\alpha_2(y)\beta_2(y)}\right] \le 0\,,\ \frac{\delta_2(y)}{\beta_2(y)} \ge 0$

Then problem **(N)** has only one solution.

The Proof:

We will first prove the uniqueness of the solution:

We suppose that the function $\boldsymbol{U(x,y) \neq const}$ in $\overline{\boldsymbol{D}}$ is a solution to the following homogeneous problem:

$$U_{xx} - U_y + C(x,y)U = 0 \qquad ;(x,y) \in D$$
$$U_{yy} - U_{xx} - \lambda_j U = 0 \qquad ;(x,y) \in D_i \quad i = 2,3\,; j = 1,2$$
$$U|_{A_0C} = 0\ ;\ U|_{BE} = 0\ ;\ U|_{y=0} = 0$$
$$U(-0,y) = \alpha_1(y)U(+0,y)$$
$$U_x(-0,y) = \beta_1(y)\,U_x(+0,y) + \delta_1(y)\,U(+0,y)$$
$$U(1+0,y) = \alpha_2(y)U(1-0,y)$$
$$U_x(1+0,y) = \beta_2(y)\,U_x(1-0,y) + \delta_2(y)U(1-0,y)$$

We will prove that the function: $\boldsymbol{U(x,y) \equiv 0}$ is a solution to this homogeneous problem and accordingly the inhomogeneous problem has a single solution.

Then for $\boldsymbol{D_1}$ the following equality is true:

17.

$$\frac{1}{2}\int_0^1 U^2(x,1)dx + \int_0^1 \tau_1^+(y)v_1^+(y)dy - \int_0^1 \tau_2^+(y)v_2^+(y)dy + \iint_{D_1}[U_x^2 + c(x,y)U^2]dxdy = 0$$

It is necessary to find the two integrals

$$I_j = \int_0^1 \tau_j^+(y)v_j^+(y)dy \quad , \quad j = 1,2$$

Then from (12) and (13) and noting the conditions (10) benefiting from the definition of the Bessel function, we find the following:

$$I_j = \int_0^1 \tau_j^+(y)v_j^+(y)dy =$$

$$= \frac{1}{\pi}\int_0^1 (1-z^2)^{-\frac{1}{2}}dz\{\frac{1}{\alpha_1(0)\beta_1(0)} \times [(\int_0^1 \cos\lambda_1 zt\,\overline{v}_1(t)dt)^2 + (\int_0^1 \sin\lambda_1 zt\,\overline{v}_1(t)dt)^2] + \int_0^1 [\frac{1}{\alpha_1(y)\beta_1(y)}][(\int_y^1 \cos\lambda_1 zt\,\overline{v}_1(t)dt)^2 + \int_y^1 \sin\lambda_1 zt\,\overline{v}_1(t)dt)^2]dy\} - \int_0^1 \frac{\delta_1(y)}{\alpha_1^2(y)\beta_1(y)}\overline{\tau}_1(y)dy.$$

$$I_2 = \int_0^1 \tau_2^+(y)v_2^+(y)dy =$$

$$= \frac{1}{\pi}\int_0^1 (1-z^2)^{-\frac{1}{2}}dz\{\frac{1}{\alpha_1(1)\beta_2(1)} \times [(\int_0^1 \cos\lambda_2 zt\,\overline{v}_2(t)dt)^2 + \int_0^1 [\frac{1}{\alpha_2(y)\beta_2(y)}] \times (\int_0^y \cos\lambda_2 zt\,\overline{v}_2(t)dt)^2 dy - \frac{1}{\alpha_2(1)\beta_2(1)} \times (\int_0^1 \sin\lambda_2 zt\,\overline{v}_2(t)dt)^2 + \int_0^1 [\frac{1}{\alpha_2(y)\beta_2(y)}] (\int_0^1 \sin\lambda_2 zt\,\overline{v}_2(t)dt)^2 dy\} - \int_0^1 \frac{\delta_2(y)}{\alpha_2^2(y)\beta_2(y)}\overline{\tau}_2(y)dy$$

Let us note that if all inequalities (15) and (16) are satisfied, then it is easy to obtain:

$$I_1 = \int_0^1 \tau_j^+(y) v_1^+(y) dy > 0$$

$$I_2 = \int_0^1 \tau_2^+(y) v_2^+(y) dy < 0$$

Then we conclude from (17) that: $U_x = 0$
So: $U(x, y) = \mu(y)$
Since: $U(0, y) = U(1, y) = 0$
We find: $\mu(y) \equiv 0$
That is: $U(x, y) \equiv 0$; $(x, y) \in \bar{D}_1$
This is on the one hand, and on the other hand and according to the oneness of solving Cauchy's problem for the system of equations (2) and (3) in squares D_2 and D_3, respectively, must be $U(x, y) \equiv 0$; $(x, y) \in \bar{D}$: This is a contradiction.
And by this we have demonstrated the validity and uniqueness of the solution.
Now let's move on to proving that the solution exists:
The solution to the first mixed problem for the parabola in open square D_1 **[3]** is given as:

$$U(x, y) = \int_0^y G_\xi(x, y, 0, \eta)\tau_1^+(\eta)d\eta - \int_0^y G_\xi(x, y, 1, \eta)\tau_2^+(\eta)d\eta + \int_0^y G(x, y, \xi, 0)\varphi(\xi)d\xi - \int_0^1 d\xi + \int_0^y C(\xi, \eta)G(x, y; \xi, \eta)U(\xi, \eta)d\eta \qquad \textbf{(18)}$$

Where:

$$G(x, y; \xi, \eta) = \frac{1}{2\sqrt{\pi(y-\eta)}} - exp\{-\frac{(x+\xi+2n)^2}{4(y-\eta)}\} \sum_{n=-\infty}^{\infty} exp\{-\frac{(x-\xi+2n)^2}{4(y-\eta)}\}$$

where $G(x, y; \xi, \eta)$ is the Greene function of the first mixed problem in square D_1.
In order to obtain the relationship between the functions
$\tau_1^+(y)$and $v_1^+(y)$ as well as the relationship between the functions $\tau_2^+(y)$ and $v_1^+(y)$ on the lines AA_0 and BB_0, the integral equation (18) must be solved using the kernel Case [9] Accordingly, it is:

$$U(x, y) = \int_0^y G_\xi(x, y, 0, \eta)\tau_1^+(\eta)d\eta - \int_0^y G_\xi(x, y, 1, \eta)\tau_2^+(\eta)d\eta + \int_0^y \Phi_1(\eta; x, y)\,\tau_1^+(\eta)d\eta$$

$$+ \int_0^y \Phi_2(\eta; x, y)\,\bar{v}_2(\eta)d\eta + \Psi(x, y) + V(x, y)$$

Where:

$$\Phi_1(\eta; x, y) = \int_\eta^y \int_0^1 G_\xi(\theta, t, 0, \eta)R_1(x, y, \theta, t)d\theta dt \quad ;$$

$$\Phi_2(\eta; x, y) = -\int_\eta^y \int_0^1 G_\xi(\theta, t, 1, \eta)R_1(x, y, \theta, t)d\theta dt \quad ;$$

$$V(x, y) = \int_0^1 G(x, y; \xi, 0)\varphi(\xi)d\xi \quad ;$$

$$V(x,y)=\int_0^y\int_0^1\int_0^1 R_1(x,y;\theta,t)G(\theta,t,\xi,0)\varphi(\xi)d\xi d\theta dt$$

Assuming that $R_1(x,y;\theta,t)$ is the nucleus of $C(\xi,\eta)G(x,y,\xi,\eta)$
let us now deduce the partial derivatives:

$$U_x|_{x=0}\equiv v_1^+(y)=\int_0^y\left\{-\frac{1}{\sqrt{\pi(y-\eta}}+\frac{2}{\sqrt{\pi(y-\eta}}\times\sum_{n=1}^{\infty}exp\,(-\frac{n^2}{y-\eta}\right\}\tau_1^+(\eta)d\eta$$

$$+\int_0^y\{\frac{1}{\sqrt{\pi(y-\eta}}exp\left[-\frac{1}{\sqrt{4(y-\eta}}\right]+\frac{1}{2\sqrt{\pi(y-\eta}}\times\sum_{n=-\infty}^{\infty}\left[exp\left\{-\frac{(-1+2\eta)^2}{4(y-\eta)}\right\}+exp\left\{-\frac{(1+2\eta)^2}{4(y-\eta)}\right\}\right]$$

$$\times\tau_2^+(\eta)d\eta+\int_0^y\phi_{1x}(\eta;0,y)\,\tau_1^+(\eta)d\eta+\int_0^y\phi_{2x}(\eta;0,y)\,\tau_2^+(\eta)d\eta+F_1(y)\qquad(19)$$

$$U_x|_{x=1}\equiv v_2^+(y)=\int_0^y\left\{-\frac{1}{\sqrt{\pi(y-\eta}}exp\left[\frac{1}{4(y-\eta)}\right]-\frac{2}{\pi(y-\eta)}\sum_{n=1}^{\infty}exp\left[-\frac{(1+2n)^2}{4(y-\eta)}\right]\right\}$$

$$\tau_1^+(\eta)d\eta+\int_0^y\{\frac{1}{2\sqrt{\pi(y-\eta)}}+\frac{1}{\sqrt{\pi(y-\eta)}}\sum_{n=1}^{\infty}exp\left(\frac{n^2}{y-\eta}\right)+\frac{1}{2\sqrt{\pi(y-\eta)}}exp\left[-\frac{1}{y-\eta}\right]$$

$$+\frac{1}{2\sqrt{\pi(y-\eta)}}\times\sum_{n=-\infty}^{\infty}exp\left[-\frac{(1+2n)^2}{y-\eta}\right]\tau_2^+(\eta)d\eta+\int_0^y\emptyset_{1x}(\eta;1,y)\tau_1^+(\eta)d\eta+$$

$$\int_0^y\emptyset_{2x}(\eta;1,y)\tau_2^+(\eta)d\eta+F_2(y)\qquad(20)$$

Where:

$$F_{i+1}(y)=\left[\frac{\partial\Psi(x,y)}{\partial x}+\frac{\partial V(x,y)}{\partial x}\right]_{x=i}\qquad;i=0,1$$

By deleting the functions $\tau_i^+(y)$ and $v_i^+(y)$ and $\tau_i^-(y)$ when $i=1,2$ of (12), (13) and (19), (20) and (21) benefiting from the conditions (10), we get a set of two integral equations:

$$v_1^-(y)+\int_0^y M_1(y,\eta)v_1^-(\eta)d\eta+\int_y^1 M_2(y,\eta)v_1^-(\eta)d\eta+\int_0^y M_3(y,\eta)v_2^-(\eta)d\eta=P_1(y);\qquad(22)$$

$$v_1^-(y)+\int_0^y M_1(y,\eta)v_1^-(\eta)d\eta+\int_y^1 M_2(y,\eta)v_1^-(\eta)d\eta+\int_y^1 M_6(y,\eta)v_1^-(\eta)d\eta=P_2(y);\qquad(23)$$

This is assuming that:

$$M_1(y,\eta)=\frac{\beta_1(y)}{\alpha_1(\eta)\sqrt{\pi(y-\eta)}}\left[1+2\sum_{n=1}^{\infty}exp\left(\frac{n^2}{y-\eta}\right)\right]+\beta_1(y)\int_0^\eta\frac{\alpha_1(t)-\alpha_1'(t)}{\alpha_1^2(t)\sqrt{\pi(y-t)}}$$

$$\left[1+2\sum_{n=1}^{\infty}exp\left(-\frac{n^2}{y-t}\right)\right]\frac{\partial}{\partial t}J_0[\lambda_1(t-\eta)]d\eta-\beta_1(y)\int_\cap^\eta J_0[\lambda_1(t-\eta)]\Phi_{1x}(t;0,y)dt;$$

$$M_2(y,\eta)=\beta_1(y)\int_0^y \frac{\alpha_1(t)-\alpha_1'(t)}{\alpha_1^2(t)\sqrt{\pi(y-t)}}\left[1+2\sum_{n=1}^{\infty} exp\left(-\frac{n^2}{y-\eta}\right)\right]\frac{\partial}{\partial t}J_0[\lambda_1(t-\eta)]d\eta$$

$$-\beta_1(y)\int_0^y J_0[\lambda_1(t-\eta)]\phi_{1x}(t;0,y)dt;-\frac{\delta_1(y)J_0[\lambda_1(t-\eta)]}{\alpha_1(y)}$$

$$M_3(y,\eta)=\frac{\beta_1(y)}{\alpha_2(\eta)\sqrt{\pi(y-\eta)}}exp\left[-\frac{1}{4(y-\eta)}\right]+\frac{1}{2}\sum_{n=-\infty}^{\infty}[exp\{-\frac{(-1+2n)^2}{4(y-\eta)}\}$$

$$+\beta_1(y)\int_\eta^y \frac{\alpha_2(t)+\alpha_2'(t)}{\alpha_2^2(t)\sqrt{\pi(y-t)}}\{exp[-\frac{1}{4(y-t)}]+exp\{-\frac{(1+2n)^2}{4(y-\eta)}\}]\}$$

$$+\sum_{n=-\infty}^{\infty}[exp\{-\frac{(-1+2n)^2}{4(y-t)}\}+exp\{-\frac{(1+2n)^2}{4(y-\eta)}\}]\}\times\frac{\partial}{\partial t}J_0[\lambda_2(t-\eta)]dt+\beta_1(y)\int_\eta^y J_0[\lambda_2(t-\eta)]$$

$$\phi_{2x}(t;0,y)dt;\frac{\beta_2(y)}{\alpha_2(\eta)\sqrt{2\pi(y-\eta)}}\{1+2\sum_{n=1}^{\infty}exp(-\frac{n^2}{y-\eta})$$

$$M_4(y,\eta)=\sum_{n=-\infty}^{\infty}exp\left[-\frac{(1+2n)^2}{4(y-\eta)}\right]+exp\left[-\frac{1}{y-\eta}\right]+\beta_2(y)\int_\eta^y\frac{\alpha_2(t)+\alpha_2'(t)}{\alpha_2^2(t)\sqrt{2\pi(y-t)}}$$

$$\left\{1+2\sum_{n=1}^{\infty}exp\left(-\frac{n^2}{y-t}\right)+exp\left[-\frac{1}{y-t}\right]+[\lambda_2(t-\eta)]dt+\sum_{n=-\infty}^{\infty}exp\left[-\frac{(1+2n)^2}{4(y-t)}\right]\right\}$$

$$\frac{\partial}{\partial t}J_0+\frac{\delta_1(y)J_0[\lambda_2(y-\eta)]}{\alpha_2(y)};\beta_2(y)\int_\eta^y J_0[\lambda_2(t-\eta)]\phi_{2x}(t;1,y)dt$$

$$\sum_{n=-\infty}^{\infty}exp[-\frac{(1+2n)^2}{4(y-\eta)}]\}\frac{\beta_2(y)}{\alpha_1(\eta)\sqrt{\pi(y-\eta)}}\{exp[-\frac{1}{y-\eta}].$$

$$M_5(y,\eta)=\beta_2(y)\int_0^\eta\frac{\alpha_1(t)+\alpha_2'(t)}{\alpha_1^2(t)\sqrt{\pi(y-t)}}\left\{exp\left[-\frac{1}{4(y-t)}\right]+\sum_{n=-\infty}^{\infty}exp\left[-\frac{(1+2n)^2}{4(y-n)}\right]\right\}$$

$$\frac{\partial}{\partial t}J_0[\lambda_1(t-\eta)]dt-\beta_2(y)\int_0^\eta J_0[\lambda_2(t-\eta)]\phi_{1x}(t;1,y)dt;$$

$$\beta_2(y)\int_0^y\frac{\alpha_1(t)+\alpha_1'(t)}{\alpha_1^2(t)\sqrt{\pi(y-t)}}exp\left[-\frac{1}{4(y-t)}\right]$$

$$M_6(y,\eta)=[\lambda_1(t-\eta)]dt-\sum_{n=-\infty}^{\infty}exp[-\frac{(1+2n)^2}{4(y-t)}]\}\frac{\partial}{\partial t}J_0\frac{\delta_1(y)J_0[\lambda_1(y-\eta)]}{\alpha_1(y)};$$

$$-\beta_2(y)\int_0^\eta J_0[\lambda_1(t-\eta)]\phi_{1x}(t;1,y)dt$$

$$P_1(y) = \beta_1(y)F_1(y) + \sigma_1(y) + \frac{\delta_1(y)[\rho_1(y) - \gamma_1(y)]}{\alpha_1(y)} + \beta_1(y)\int_0^y \frac{\rho_1(t) - \gamma_1(t)}{\alpha_2(t)} + \phi_{1x}(t;1,y) +$$

$$\phi_{2x}(t;0,y)\}dt - \beta_1(y)\int_0^y \frac{1}{\sqrt{\pi(y-n)}} + \frac{\rho_2(t) - \gamma_2(t)}{\alpha_2(t)} \times \{\frac{\alpha_1(\eta)\rho_1'(\eta) - \alpha_1'(\eta)\rho_1(\eta) + \alpha_1'(\eta)\gamma_1(\eta) - \alpha_1(\eta)\gamma_1'(\eta)}{\alpha_1^2(\eta)}$$

$$\times [1 + 2\sum_{n=1}^{\infty} exp\,(-\frac{n^2}{y-\eta}]\}d\eta + \beta_1(y)\int_0^y \frac{1}{2\times\sqrt{\pi(y-\eta)}} \times$$

$$\frac{\alpha_2(\eta)\rho_2'(\eta) - \alpha_2'(\eta)\rho_2(\eta) + \alpha_2'(\eta)\gamma_2(\eta) - \alpha_2(\eta)\gamma_2'(\eta)}{\alpha_2^2(\eta)}$$

$$\times \sum_{n=-\infty}^{\infty} [exp\,\{-\frac{(-1+2n)^2}{4(y-c)}\} + \{exp[-\frac{1}{4(y-\eta)} + exp\{-\frac{(1+2n)^2}{4(y-\eta)}\}]\}d\eta \quad ;$$

$$P_2(y) = \beta_2(y)F_2(y) + \sigma_2(y) + \beta_2(y)\int_0^y \frac{\sigma_1(t) + \gamma_1(t)}{\alpha_1(t)} \times \phi_{1x}(t;1,y) + \phi_{2x}(t;1,y)]dt -$$

$$\beta_2(y)\int_0^y \frac{1}{\sqrt{\pi(y-\eta)}} \frac{\rho_2(t) - \gamma_2(t)}{\alpha_2(t)} \times \frac{\alpha_1(\eta)\rho_1'(\eta) - \alpha_1'(\eta)\rho_1(\eta) + \alpha_1'(\eta)\gamma_1(\eta) - \alpha_1(\eta)\gamma_1'(\eta)}{\alpha_1^2(\eta)}$$

$$\times \beta_2(y)\int_0^y \frac{1}{2\times\sqrt{\pi(y-\eta)}} \times \sum_{n=-\infty}^{\infty} exp\,[-\frac{(1+2n)^2}{4(y-c)}]\}d\eta + x\{exp\left[-\frac{1}{4(y-\eta)}\right] +$$

$$\sum_{n=-\infty}^{\infty} exp\,[-\frac{(1+2n)^2}{4(y-\eta)}]\}d\eta\, exp[-\frac{1}{(y-\eta)}] + \{1 + 2\sum_{n=1}^{\infty} exp\left(-\frac{n^2}{y-\eta}\right) \;.$$

By solving equation (23) for the function $\overline{\mathbf{v}}_2(y)$, we find:

$$\int_y^1 M_8\,(y,\eta)\overline{\mathbf{v}}_1(\eta)d\eta \qquad (24) \qquad \overline{\mathbf{v}}_2(y) = P_3(y) - \int_0^y M_7\,(y,\eta)\overline{\mathbf{v}}_1(\eta)d\eta$$

Where:

$$\int_0^\eta M_6\,(t,\eta)\mathrm{R}_2(y,t)dt;\;\; M_7(y,\eta) = M_5(y,\eta) + \int_\eta^y M_5\,(t,\eta)\mathrm{R}_2(y,t)dt + M_8(y,\eta) = M_6(y,\eta) +$$

$$\int_0^y M_6\,(t,\eta)\mathrm{R}_2(y,t)dt; P_3(y) = P_2(y) + \int_0^y P_2\,(\eta)R_2(y,\eta)d\eta;$$

This is assuming that $R_2(y,\eta)$ is the state nucleus of $M_4(y,\eta)$.

If we assume:

$$k(y,\eta) = \begin{cases} k_1(y,\eta) & if \quad 0 \le \eta \le y \\ k_2(y,\eta) & if \quad y < \eta \le 1 \end{cases}$$

Substituting (24) into (22), we get the Friedholm integral equation of the second type with respect to the function $\overline{v}_1(y)$:

$$\overline{v}_1(y) + \int_0^1 k\,(y,\eta)\overline{v}_1(\eta)d\eta = P(y)$$

Where:

$$k_1(y,\eta) = \mathrm{M}_1(y,\eta) - \int_{\eta}^{y} M_3\,(y,t)M_7(t,\eta)dt - \int_{\eta}^{y} M_3\,(y,t)M_8(t,\eta)dt;$$

$$k_2(y,\eta) = \mathrm{M}_2(y,\eta) - \int_{\eta}^{y} M_3\,(y,t)M_8(t,\eta)dt;\ P(y) = P_1(y) - \int_{\eta}^{y} M_3\,(y,t)P_3(t)dt.$$

Hence, based on Friedholm's hypothesis in the integrative equations and based on the proof of the oneness of the solution, the integrative equation (25) has a single continuous solution. Then from the relations (10), (12), (13), (23), (25) the functions are determined in a unique way

$\upsilon_2^-(y), \upsilon_1^-(y), \upsilon_2^+(y), \upsilon_1^+(y), \tau_2^-(y), \tau_1^-(y), \tau_2^+(y), \tau_1^+(y)$

Thus, we get the solution to the problem in square D_1. The solution in squares D_2 and D_3 is done similarly to solving the Cauchy problem in the xoy plane.

Thus, we have proven that there is a solution to the issue at hand.

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