مجـلـة الـتربـوي
Journal of Educational
ISSN: 2011- 421X
Arcif Q3

معامل التأثير العربي 1.5
العدد 19

مجلة التربوي
مجلة علمية محكمة تصدر عن كلية التربية
جامعة المرقب

العدد التاسع عشر
يوليو 2021م

هيئـــة تحرير
مجلة التربوي

- المجلة ترحب بما يرد عليها من أبحاث وعلى استعداد لنشرها بعد التحكيم .
- المجلة تحترم كل الاحترام آراء المحكمين وتعمل بمقتضاها .
- كافة الآراء والأفكار المنشورة تعبر عن آراء أصحابها ولا تتحمل المجلة تبعاتها .
- يتحمل الباحث مسؤولية الأمانة العلمية وهو المسؤول عما ينشر له .
- البحوث المقدمة للنشر لا ترد لأصحابها نشرت أو لم تنشر .

مجـلة الـتربـوي
Journal of Educational
ISSN: 2011- 421X
Arcif Q3

معامل التأثير العربي 1.5
العدد 19

ضوابط النشر:

يشترط في البحوث العلمية المقدمة للنشر أن يراعى فيها ما يأتي :

- أصول البحث العلمي وقواعده .
- ألا تكون المادة العلمية قد سبق نشرها أو كانت جزءا من رسالة علمية .
- يرفق بالبحث تزكية لغوية وفق أنموذج معد .
- تعدل البحوث المقبولة وتصحح وفق ما يراه المحكمون .
- التزام الباحث بالضوابط التي وضعتها المجلة من عدد الصفحات ، ونوع الخط ورقمه ، والفترات الزمنية الممنوحة للتعديل ، وما يستجد من ضوابط تضعها المجلة مستقبلا .

تنبيهات :

- للمجلة الحق في تعديل البحث أو طلب تعديله أو رفضه .
- يخضع البحث في النشر لأولويات المجلة وسياستها .
- البحوث المنشورة تعبر عن وجهة نظر أصحابها ، ولا تعبر عن وجهة نظر المجلة .

Information for authors

1- Authors of the articles being accepted are required to respect the regulations and the rules of the scientific research.
2- The research articles or manuscripts should be original and have not been published previously. Materials that are currently being considered by another journal or is a part of scientific dissertation are requested not to be submitted.
3- The research articles should be approved by a linguistic reviewer.
4- All research articles in the journal undergo rigorous peer review based on initial editor screening.
5- All authors are requested to follow the regulations of publication in the template paper prepared by the editorial board of the journal.

Attention

1- The editor reserves the right to make any necessary changes in the papers, or request the author to do so, or reject the paper submitted.
2- The research articles undergo to the policy of the editorial board regarding the priority of publication.
3- The published articles represent only the authors' viewpoints.

مجلة التربوي
Journal of Educational
ISSN: 2011- 421X
Arcif Q3

معامل التأثير العربي 1.5
العدد 19

λ-Generalizations And ɡ- Generalizations

Khadiga Ali Arwini[1, *], Entisar Othman Laghah[2]
[1]Mathematics Department, Tripoli University, Tripoli-Libya
[2]Mathematics Department, Zawia University, Zawia-Libya
Kalrawini@yahoo.com*

ABSTRACT

In this paper, we use the concept of λ-closed set and ɡ-closed set to define two classes of generalized regular closed sets; namely λ-generalizations and ɡ-generalizations. The class of λ-generalizations includes: r λ-closed set, $r^*λ$-closed set and r $λ^*$-closed set, while the second class of generalizations includes: ɡ.r-closed set, ɡ.r-closed set and r.$ɡ^*$-closed set. We investigate the characterizations of these generalizations, moreover, we illustrate the implications of these classes among themselves and with the known sets, and finally we study their behavior in regular spaces and in extremely disconnected spaces.

Keywords: *Topological space and generalizations, regular space, extremely disconnected space.*

AMS Subject Classification (2000): 54A05, 54D10, 54G05.

1. INTRODUCTION

The concept of regular closed sets was introduced be Stone in 1937 [1], where a subset in a topological space is called regular closed (briefly r-closed) if its equals to the closure of its interior, Stone studied this class of sets, and showed that r-closed set is stronger than closed set. The family of r-closed sets has some applications in the semiregularization space [1,2], also in a generalization for algebraic openings and closings in a complete lattice [3].

Many studies in the literature have been made on defining different generalizations of closed sets as; v-sets, ɡ-closed sets, λ-closed sets, α-closed sets, semi-closed sets, preclosed sets, b-closed sets, etc., where these notions were defined using the closure and the interior operations. The concept of these generalizations play a significant role in general topology, and used to derive several forms of higher and lower separation axioms and compactness.

Maki [4] introduced the notion of ʌ-sets in topological spaces, where a ʌ-set is a set that equals to its kernel, i.e. to the intersection of all open supersets of the set, then in 2021, Almarghani and Arwini [5] introduced generalizations of regular closed sets, namely v_r-sets, g.v_r-sets, g^*.v-set and g^*.v_r-set, by considering the notion of the closure operator $ʌ_r$-closure. Arenas et al. [6] introduced and investigated the notion of λ-closed sets by involving ʌ-sets and closed sets, this enabled them to obtain new separation axioms by utilizing the notion of λ-closure operator. The concept of generalized closed set (briefly g-closed) was due to Levine in 1970 [7], when he used this notation to define a space called $T_{\frac{1}{2}}$-space, and he showed that $T_{\frac{1}{2}}$ is strictly between the spaces T_1 and T_0 [8,9,10]. In 1993, Palaniappan [11] introduced the concept of regular generalized closed sets (briefly r.g-closed) and he proved that this class of sets is weaker than the class of g-closed sets. Later on 2011 [12] Bhattacharya defined a new class of sets called generalized regular closed sets (briefly g.r-closed), when he studied the behavior relative to unions, intersections and subspaces; moreover, he proved that these class of sets are weakly ordered as; r-closed set, g.r-closed set, g-closed set then r.g-closed set.

The purpose of this article is to use the notions of λ-closed set and g-closed set to define two classes of generalized regular closed sets; namely λ-generalizations and g-generalizations, where λ-generalizations class consists the sets: rλ-closed sets, $r^*\lambda$-closed sets and $r\lambda^*$-closed sets, while the second class of generalizations contains the sets: g.r-closed sets, g.r-closed sets and $r.g^*$-closed sets, where g.r-closed sets and r.g-closed sets were due to Bhattacharya and Palaniappan, as we mentioned before. We discuss the properties of these generalizations, moreover, we illustrate the implications of these classes among themselves and with the known sets, and finally we investigate their behavior in regular spaces and in extremely disconnected spaces.

We divided our article into five main sections as; introduction, preliminaries, λ-generalizations, g-generalizations and finaly conclusion.

2. PRELIMINARIES

In this section, we recall the definitions of regular-closed sets, v-sets, λ-closed sets and g-closed sets, with some of their properties that we need in the sequel.

Throughout this paper (X,τ) represented non-empty topological space, and will replaced by X if there is no chance of confusion, no separation axioms assumed unless otherwise mentioned. If A is a subset of a space X, the notions $\overline{A}$ and A^o denote the closure and the interior of A; respectively.

2. 1. Regular Closed Sets

Definition 2.1.1. [1] A subset B of a space (X,τ) is called regular closed (briefly r-closed) if $B=\overline{B^\circ}$, while the set B is called δ-closed set if B is the intersection of r-closed sets. The family of all r-closed sets in (X,τ) is denoted by $RC(X,\tau)$.

Corollary 2.1.1. [1]

1- Every r-closed set is δ-closed set.
2- Every δ-closed set is closed set.
3- Intersection of r-closed sets is not necessarily r-closed.
4- Finite union of r-closed sets is r-closed.

Definition 2.1.2. [13] Let A be a subset of X then, the r-closure of A is defined as the intersection of all r-closed sets containing A, and is denoted $\overline{A}^r$.

Proposition 2.1.1. [13] Let X be a space and A, B⊆X, then:

1- $\overline{A}^r$ is δ-closed set but not r-closed set in general.
2- $A \subseteq \overline{A} \subseteq \overline{A}^r$.
3- If A is r-closed then $A = \overline{A}^r$.
4- A is δ-closed if and only if $A = \overline{A}^r$.

Definition 2.1.3. [13] A space X is called regular-space if for any closed set F and x∉F there exist disjoint open sets U and V such that x∈U and F⊆V.

Definition 2.1.4. [14] A space X is called extremely disconnected (briefly e.d) if, the closue of every open set in x is also open.

Proposition 2.1.2. [14] In extremely disconnected space (X, τ); we have:

1- Any r-closed set is clopen.
2- Any r-open set is clopen.
3- $RO(X,\tau) = RC(X,\tau) = \tau \cap \mathcal{F}$, where $RO(X,\tau)$ is the family of all r-open sets in X, and $\mathcal{F}$ is the collection of all closed sets in X.

2. 2. ʌ-Sets and Regular ʌ-Sets

Definition 2.2.1. [4] Let B be a subset of a topological space (X, τ), then:

1- $B^{\vee} = \cup\{F: F \subseteq B, F \text{ is closed}\}$.
2- $B^{\wedge} = \cap\{U: B \subseteq U, U \text{ is open}\}$.
3- $B^{\vee r} = \cup\{N: N \subseteq B, N \text{ is r-closed}\}$.
4- $B^{\wedge r} = \cap\{W: B \subseteq W, W \text{ is r-open}\}$.

Definition 2.2.2. [4] A subset B of a topological space (X, τ) is called:

1- v-set if $B^{\vee} = B$.
2- ʌ-set if $B = B^{\wedge}$.
3- $\vee_r$-set if $B^{\vee r} = B$.
4- $\wedge_r$-set if $B = B^{\wedge r}$.

Theorem 2.2.1. [5] Let A and B be subsets of a topological space (X, τ), then the following properties are hold:

1- $B^{\vee r} \subseteq B^{\vee} \subseteq B \subseteq B^{\wedge} \subseteq B^{\wedge r}$..
2- If A⊆B then $A^{\vee r} \subseteq B^{\vee r}$ and $A^{\wedge r} \subseteq B^{\wedge r}$.
3- $(B^{\vee r})^{\vee r} = B^{\vee r}$.
4- $(B^{\wedge r})^{\wedge r} = B^{\wedge r}$.
5- $(B^{\vee r})^c = (B^c)^{\wedge r}$.
6- $(B^{\wedge r})^c = (B^c)^{\vee r}$.

Theorem 2.2.2. [5] In a topological space (X, τ) the following hold:

1- Every r-closed is v_r-set.
2- Every v_r-set is v-set.
3- Every r-open is Λ_r-set.
4- Every Λ_r- set is Λ-set.
5- B is v_r-set iff B^c is Λ_r-set.

Diagram 1, shows the implications between the generalizations:

$$\begin{array}{ccc} \text{r-closed} & \Longrightarrow & v_r\text{-set} \\ \Downarrow & & \Downarrow \\ \text{closed} & \Longrightarrow & v\text{-set} \end{array} \quad \text{and} \quad \begin{array}{ccc} \text{r-open} & \Longrightarrow & \Lambda_r\text{-set} \\ \Downarrow & & \Downarrow \\ \text{open} & \Longrightarrow & \Lambda\text{-set} \end{array}$$

Diagram 1. Generalizations of regular closed sets and regular open sets.

Theorem 2.2.3. [5] In e.d space (X, τ), if $A \subseteq X$ then $A^{\wedge r} = \overline{A}^r$.

Theorem 2.2.4. [5] In regular space (X, τ), if $A \subseteq X$, then $\overline{A}^r = \overline{A}$.

Corollary 2.2.1. [5] In regular e.d space X, if $A \subseteq X$, then $A^{\wedge r} = \overline{A}^r = \overline{A}$.

2. 3. λ-Closed Sets

Definition 2.3.1. [6] A subset A of a topological space (X, τ) is called λ-closed if $A = L \cap F$, where L is Λ-set and F is closed set.

Corollary 2.3.1. [6]
1-Every closed set is λ-closed
2- Every Λ-set is λ-closed.

Theorem 2.3.1. [6] For a subset A of a topological space (X, τ) the following statements are equivalent:
1- A is λ-closed.
2- $A = A^{\wedge} \cap \overline{A}$.

2. 4. g-Closed Sets

Definition 2.4.1. [10,11,12] A subset A of a topological space (X, τ) is said to be :
1- Generalized closed (briefly g-closed) if $\overline{A} \subseteq U$ whenever $A \subseteq U$ and U is open in X.
2- Regular generalized closed (briefly r.g-closed) if $\overline{A} \subseteq W$ whenever $A \subseteq W$ and W is r-open .

3- Generalized regular closed (briefly g.r-closed) if $\overline{A}^{r} \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Corollary 2.4.1. [10]
1- Any closed set is g-closed.
2- Union of g-closed sets is g-closed.
3- Finite Intersection of g-closed sets is not necessarily g-closed.

Corollary 2.4.2. [10] If X is a topological space and x is a point in X such that {x} is not closed, then $\{x\}^c$ is a g-closed set.

Theorem 2.4.1. [10] For a subset A of a topological space (X, τ) the following statements are equivalent:
1- A is closed.
2- A is g-closed and λ-closed.

3. λ-GENELAIZATIONS

In this section, we define a new class of generalizations by involving Λ_r-sets and r-closed sets; namely λ-generalizations that contains the sets: r λ-closed set, $r^*\lambda$-closed set and r λ^*-closed set. We prove that these sets are weakly ordered as: r-closed set, r λ-closed set, r λ^*-closed set, λ-closed set. In addition, we investigate the properties of this class of generalizations in regular spaces and in e.d spaces.

Definition 3.1. A subset A of a topological space (X, τ) is called:
1- r λ-closed if $A=L\cap F$, where L is Λ_r-set and F is r-closed set.
2- r λ^*-closed if $A=L\cap F$, where L is Λ-set and F is r-closed set.
3- $r^*\lambda$-closed if $A=L\cap F$, where L is Λ_r-set and F is closed set.

Theorem 3.1. In a topological space (X, τ), we have:
1- Any r-closed set in X is r λ-closed and r λ^*-closed.
2- Any closed set in X is $r^*\lambda$-closed and λ-closed.
3- Every Λ_r-set is r λ-closed set and $r^*\lambda$-closed.
4- Every Λ-set is r λ^*-closed set and λ-closed.

Corollary 3.1.
1- Every r λ-closed set is r λ^*-closed and $r^*\lambda$-closed.
2- Every r λ^*-closed is λ-closed.
3- Every $r^*\lambda$-closed is λ-closed.
Proof:
1- Direct since every r-closed is closed and Λ_r-set is Λ-set.
2- Direct since every r-closed is closed
3- Direct since every Λ_r-set is Λ-set.

Example 3.1. Let X= {a,b,c,d} and τ= {Ø, X, {a}, {b,c}, {a,b,c}}, then RC(X, τ)= {Ø, X, {b,c,d}, {a,d}}. If A= {a,b,c}, B= {b,c} and C= {d}, then:

3- A is r λ^*-closed but not r λ-closed; also A is λ-closed but not $r^*\lambda$-closed.

4- B is r λ-closed but not r-closed; also B is $r^*\lambda$-closed but not closed.

5- C is λ-closed but not r λ^*-closed; also C is $r^*\lambda$-closed but not r λ-closed.

Theorem 3.2. A subset A of a topological space (X, τ) is r λ-closed iff $A=A^{\Lambda r} \cap \overline{A}^r$.

Proof: ⇒ Suppose A is r λ-closed then A=B∩F, where B is Λ_r-set and F is r-closed set, so $A\subseteq \overline{A}^r$ and $A\subseteq A^{\Lambda r}$, thus $A\subseteq A^{\Lambda r} \cap \overline{A}^r$ …..(1), A=B∩F⊆F then $\overline{A}^r\subseteq F$ (F is r-closed), A= B∩F⊆B then $A^{\Lambda r}\subseteq B$ (B is Λ_r -set) and $A^{\Lambda r} \cap \overline{A}^r \subseteq B \cap F$, i.e. $A^{\Lambda r} \cap \overline{A}^r \subseteq A$… (2). From (1) and (2) we get $A=A^{\Lambda r} \cap \overline{A}^r$.

⇐ Suppose $A=A^{\Lambda r} \cap \overline{A}^r$, since $A^{\Lambda r}$ is Λ_r-set and $\overline{A}^r$ is r-closed then A is r λ-closed.

Theorem 3.3. A subset A of a topological space (X, τ) is r λ^*-closed iff $A=A^{\wedge} \cap \overline{A}^r$.

Proof: ⇒ Suppose A is r λ^*-closed, so A= B∩F where B is Λ-set and F is r-closed set, and since $A\subseteq \overline{A}^r$ and $A\subseteq A^{\wedge}$ we have $A \subseteq A^{\wedge} \cap \overline{A}^r$ ......(1), now since A=B∩F⊆F, then $\overline{A}^r\subseteq F$ (F is r-closed), A=B∩F⊆ B then $A^{\wedge} \subseteq B$ (B is Λ-set), we obtain $A^{\wedge} \cap \overline{A}^r \subseteq B \cap F$, i.e. $A^{\wedge} \cap \overline{A}^r \subseteq$ A……(2). From (1) and (2) we get $A=A^{\wedge} \cap \overline{A}^r$.

⇐ Suppose $A=A^{\wedge} \cap \overline{A}^r$, since $A^{\wedge}$ is Λ-set and $\overline{A}^r$ is r-closed then A is $r^*\lambda$-closed.

Theorem 3.4. A subset A of a topological space (X, τ) is $r^*\lambda$ -closed iff $A=A^{\Lambda r} \cap \overline{A}$.

Proof: ⇒ Suppose A is $r^*\lambda$-closed, so A= T∩C, where T is Λ_r-set and C is closed set, since $A\subseteq \overline{A}$ and $A\subseteq A^{\Lambda r}$ then $A\subseteq A^{\Lambda r} \cap \overline{A}$ ......(1), now A=T∩C⊆C then $\overline{A}\subseteq C$ (C is closed), A= T∩C⊆ T so $A^{\Lambda r} \subseteq T$ (T is Λ_r-set), we have $A^{\Lambda r} \cap \overline{A}\subseteq T \cap C$, i.e. $A^{\Lambda r} \cap \overline{A}\subseteq A$……(2). From (1) and (2) we get $A=A^{\Lambda r} \cap \overline{A}$.

⇐ Suppose $A=A^{\Lambda r} \cap \overline{A}$, since $A^{\Lambda r}$ is Λ_r-set and $\overline{A}$ is closed then A is $r^*\lambda$ -closed.

Theorem 3.5. In regular space (X, τ) if A⊆X then:

1- A is r λ^*-closed iff A is λ-closed.

2- A is r λ-closed iff A is $r^*\lambda$-closed.

Proof:

1- ⇒ Direct.

⇐ If A is λ-closed then from theorems (2.3.1), (2.2.4) and (3.3) we obtain $A= A^{\wedge} \cap \overline{A}=A^{\wedge} \cap \overline{A}^r$, so A is r λ^*-closed.

2- ⇒ Direct.

⇐ If A is $r^*\lambda$-closed then from theorems (3.4), (2.2.4) and (3.2) we obtain $A= A^{\Lambda r} \cap \overline{A}=A^{\Lambda r} \cap \overline{A}^r$, so A is r λ-closed.

Theorem 3.6. In e.d space (X, τ) if $A \subseteq X$ then:

1- A is r λ-closed iff $\overline{A}^r = A$.
2- A is r λ*-closed iff A is ʌ-set.
3- A is r*λ-closed iff A is closed.

Proof:

1- ⇒ If A is r λ-closed, then $A = A^{\Lambda r} \cap \overline{A}^r = \overline{A}^r \cap \overline{A}^r = \overline{A}^r$.
⇐ If $\overline{A}^r = A$, then $A^{\Lambda r} \cap \overline{A}^r = A^{\Lambda r} \cap A = A$, so A is r λ-closed.

2- ⇒ If A is r λ*-closed, then $A = A^{\wedge} \cap \overline{A}^r = A^{\wedge} \cap A^{\Lambda r} = A^{\wedge}$, so A is ʌ-set.
⇐ If A is ʌ-set, then $A^{\wedge} \cap \overline{A}^r = A \cap \overline{A}^r = A$, so A is r λ*-closed.

3- ⇒ If A is r*λ-closed, then $A = A^{\Lambda r} \cap \overline{A} = \overline{A}^r \cap \overline{A} = \overline{A}$, so A is closed.
⇐ If A is closed, then $A^{\Lambda r} \cap \overline{A} = A^{\Lambda r} \cap A = A$, so A is r*λ-closed.

Corollary 3.2. In e.d regular space X, if $A \subseteq X$, then these statements are equivalent:

1- A is ʌ-set.
2- A is λ-closed.
3- A is r λ*-closed.

Proof: Direct from the previous theorems.

Corollary 3.3. In e.d regular space X, if $A \subseteq X$, then these statements are equivalent:

1- A is r λ-closed.
2- A is r*λ-closed.
3- A is closed.
4- $\overline{A}^r = A$.

Proof: Direct from theorems (3.5) and (3.6).

4. g-GENERALIZATIONS

In the present section, we define a new generalization of r-closed sets; namely r.g*-closed sets. We study their properties and illustrate the implication of this set with the known sets; as is g-closed set, r.g-closed set and g.r-closed set, then we investigate the behaviour of these generalizations in regular spaces and in e.d spaces.

Definition 4.1. A subset A of a space X is said to be regular generalized star closed (briefly r.g*-closed) if $\overline{A}^r \subseteq W$ whenever $A \subseteq W$ and W is r-open in X.

Corollary 4.1. In any a topological space (X, τ) these statements are hold:

1- Every r-closed set is g.r-closed.
2- Every g.r-closed set is r.g*-closed.
3- Every r.g*-closed set is r.g-closed.
4- Every g-closed set is r.g-closed.

Proof:

1- Direct since $\overline{A}^r = A$

2- Direct since every r-open set is open.

3- Direct since $\overline{A} \subseteq \overline{A}^r$ for any subset A in X.

4- Direct since every r-open set is open.

Examples 4.1. In the cofinite topology on ℝ, we have RO(ℝ ,τ_c) ={ℝ, ∅}. If A is any non-empty finite set then $\overline{A}^r$ = ℝ and the only r-open set W such that A⊆W is W= ℝ, so A is r.g⋆-closed, but not g.r-closed since $\overline{A}^r$ = ℝ and U={x}^{c}, where x∉ A is an open set contains A, but $\overline{A}^r$ ⊈U.

Corollary 4.2. If X is a topological space and x is a point in X such that {x} is not r-closed, then $\{x\}^c$ is r.g-closed set.

Theorem 4.1. A subset A in a topological space (X,τ) is g-closed iff $\overline{A} \subseteq A^{\wedge}$.

Proof: ⇒ Let A be a g-closed set in X , and let V be an open set such that A⊆V , then A⊆ $A^{\wedge}$ ⊆V, since A is g-closed and V is open then $\overline{A}$ ⊆V**,** i.e. for any open set V such that A ⊆V we have $\overline{A}$ ⊆V, so $\overline{A} \subseteq \bigcap_{A\subseteq V} V$. Hence $\overline{A} \subseteq A^{\wedge}$.

⇐ Suppose $\overline{A} \subseteq A^{\wedge}$and V is an open set such that A ⊆V, then A⊆ $A^{\wedge}$⊆V, so $\overline{A} \subseteq A^{\wedge}$⊆V, i.e. A is g-closed set**.**

Theorem 4.2. A subset A in a topological space (X,τ) is g.r-closed iff $\overline{A}^r \subseteq A^{\wedge}$.

Proof: ⇒ For any open set U in X such that A ⊆U we have A⊆ $A^{\wedge}$⊆U and $\overline{A}^r$ ⊆U since A is g.r-closed so $\overline{A}^r \subseteq \bigcap_{A\subseteq V} U$. U is an open set, then $\overline{A}^r \subseteq A^{\wedge}$.

⇐ Suppose $\overline{A}^r \subseteq A^{\wedge}$, and U is an open set in X such that A ⊆U, then A⊆ $A^{\wedge}$⊆U we have $\overline{A}^r \subseteq A^{\wedge}$⊆U then $\overline{A}^r$ ⊆U. Hence A is g.r-closed.

Theorem 4.3. A subset A in a topological space (X,τ) is r.g⋆-closed iff $\overline{A}^r \subseteq A^{\wedge_r}$.

Proof: ⇒ Suppose A is r.g⋆-closed set, then for any r-open set W in X such that A ⊆W we have A⊆$A^{\wedge_r}$⊆W since A is r.g⋆-closed, we get $\overline{A}^r$ ⊆W, then $\overline{A}^r \subseteq \bigcap_{A\subseteq w} W$, W is r-open so $\overline{A}^r \subseteq A^{\wedge_r}$.

⇐ Suppose $\overline{A}^r \subseteq A^{\wedge_r}$ and W is r-open set such that A ⊆W then A⊆$A^{\wedge_r}$⊆W so $\overline{A}^r \subseteq A^{\wedge_r}$⊆W, i.e. $\overline{A}^r$ ⊆W hence A is r.g⋆-closed.

Corollary 4.3. For a subset A of a topological space (X, τ), we have:

1- If A is r-closed set then A is r.g⋆-closed and r λ-closed.
2- If A is r.g⋆-closed and r λ-closed then A is δ-closed.

Proof:

1- Direct from theorem (3.1) and corollary (4.1).

2- Since A is $r.\mathfrak{g}^{\star}$-closed and r λ-closed, we have $\overline{A}^{r} \subseteq A^{\Lambda_r}$ and $A = A^{\Lambda_r} \cap \overline{A}^{r}$, then $\overline{A}^{r} \subseteq A^{\Lambda_r} \cap \overline{A}^{r}$, so $\overline{A}^{r} \subseteq A$, i.e. $\overline{A}^{r} = A$. We get A is δ-closed closed.

Corollary 4.4. In e.d space (x, τ), any subset of X is $r.\mathfrak{g}$-closed and $r.\mathfrak{g}^{\star}$-closed.
Proof: Let A⊆X, and W is r-open set such that A⊆W, since any r-open set in e.d space is r-closed, then $\overline{A}^{r} \subseteq \overline{W}^{r}$ so $\overline{A}^{r} \subseteq W$ and we have $\overline{A} \subseteq \overline{A}^{r} \subseteq W$, thus A is $r.\mathfrak{g}$-closed and $r.\mathfrak{g}^{\star}$-closed.

Corollary 4.5. In regular space (X, τ), a subset A of X is $r.\mathfrak{g}$-closed iff A is $r.\mathfrak{g}^{\star}$-closed.
Proof: ⇐ Direct since any $r.\mathfrak{g}^{\star}$-closed is $r.\mathfrak{g}$-closed.
⇒ Suppose A is $r.\mathfrak{g}$-closed and A⊆W when W is r-open set, then $\overline{A} \subseteq W$ since X is regular, we have $\overline{A}^{r} = \overline{A}$, so $\overline{A}^{r} \subseteq W$, hence A is $r.\mathfrak{g}^{\star}$-closed.

Corollary 4.6. In regular space (X, τ), any subset A of X is $\mathfrak{g}$-closed iff A is $\mathfrak{g}.r$-closed.
Proof: ⇐ Direct.
⇒ Suppose A is $\mathfrak{g}$-closed, and A⊆U where U is an open set, then $\overline{A} \subseteq U$, since X is regular space $\overline{A}^{r} = A$ so $\overline{A}^{r} \subseteq \overline{A} \subseteq U$, thus A is $\mathfrak{g}.r$-closed.

CONCLUSION

In this paper, we introduce new classes of generalizations using the notions of λ-closed sets and $\mathfrak{g}$-closed sets; namely λ-generalizations and $\mathfrak{g}$-generalizations. The first class of generalizations includes; r λ-closed set, $r^{*}\lambda$-closed set and r λ^{*}-closed set, while the class of $\mathfrak{g}$-generalizations includes; $\mathfrak{g}.r$-closed set, $\mathfrak{g}.r$-closed set and $r.\mathfrak{g}^{*}$-closed set, where $\mathfrak{g}.r$-closed and $\mathfrak{g}.r$-closed were due to Bhattacharya and Palaniappan. The characterizations of these generalizations are studied, moreover, we illustrate the implications between these sets, and finally we study their behavior in regular spaces and in extremely disconnected spaces.

Here we summarize our results:

- The following diagrams show the implications between the new classes of generalizations:

$$\begin{array}{ccccc} \text{r-closed} & \Rightarrow & r\,\lambda\text{-closed} & \Rightarrow & r\,\lambda^{*}\text{-closed} \\ \Downarrow & & \Downarrow & & \Downarrow \\ \text{Closed} & \Rightarrow & r^{*}\lambda\text{-closed} & \Rightarrow & \lambda\text{-closed} \end{array}$$

Diagram 2. Implication between the class of λ-generalization sets.

$$\begin{array}{ccccc} \text{r-closed} & \Longrightarrow & \mathfrak{g}.\text{r-closed} & \Longrightarrow & \text{r}.\mathfrak{g}^{\star}\text{-closed} \\ \Downarrow & & \Downarrow & & \Downarrow \\ \text{Closed} & \Longrightarrow & \mathfrak{g}\text{-closed} & \Longrightarrow & \text{r}.\mathfrak{g}\text{-closed} \end{array}$$

Diagram 3. Implication between the class of ɡ-generalization sets.

- In any space X, if a singleton {x} is not r-closed then $\{x\}^c$ is r.ɡ-closed set.
- In any space X, if A⊆X then:
- A is ɡ-closed iff $\overline{A} \subseteq A^{\wedge}$.
- A is ɡ.r-closed iff $\overline{A}^{r} \subseteq A^{\wedge}$.
- A is r.ɡ*-closed iff $\overline{A}^{r} \subseteq A^{\wedge_r}$.
- If A is r. ɡ*-closed and r λ-closed then A is δ-closed.
- In regular space X, if A⊆X then:
- $\overline{A}^{r} = \overline{A}$.
- A is λ-closed set iff A is r λ*-closed.
- A is r λ-closed set iff A is r*λ -closed.
- A is ɡ-closed-set iff A is ɡ. r-closed.
- A is r. ɡ-closed-set iff A is r. ɡ*-closed.
- In e.d space X, if A⊆X then:
- $\overline{A}^{r} = A^{\wedge_r}$.
- A is closed iff A is r*λ-closed.
- A is ʌ-set iff A is r λ*-closed.
- $\overline{A}^{r}$ =A iff A is r λ-closed.
- Any subset of X is r. ɡ-closed and r. ɡ*-closed.
- In regular e.d space X, if A⊆X then these statements are equivalent:
- A is ʌ-set.
- A is λ-closed.
- A is r λ*-closed.
- In regular e.d space X, if A⊆X then these statements are equivalent:
- A is r λ-closed.
- A is r*λ-closed.
- A is closed.
- $\overline{A}^{r}$=A.

References

[1] M. H. Stone, Applications Of The Theory Of Boolean Rings To General Topology. Trans. Am. Math. Soc., 41(1937) 375-481.

[2] Hisham Mahdi, Fadwa Nasser. On Minimal and Maximal Regular Open Sets, Mathematics and Statistics 5 (2) (2017) 78-83.

[3] C. Ronse. Regular Open or Closed Sets, Philips Research Laboratory Brussels, A v. E van Beclaere 2 B (1990) 1170.

[4] H. Maki, Generalized ʌ-Sets and The Associated Closure Operator. The Special Issue in Commemoration of Prof. Kazusada IKEDA's Retirement, 1 (1986) 139–146.

[5] Mabrouka Almarghani and K. A. Arwini, Generalizations of Regular Closed Set. An International Scientific Journal 152 (2021) 55-68

[6] F.G. Arenas, J. Dontchev and M.L. Puertas, Unification approach to the separation axioms between T_0 and completely hausdorff. Acta Math. Hungar (2000).

[7] N. Levine, Generalized Closed Sets In Topology. Rend. Circ. Mat. Palermo, 19 (2) (1970) 89-96.

[8] N. Levine A decomposition of continuity in topological spaces, Amer. Math. Monthly, 68 (1961), 44–46.

[9] Levine, N and W. Dunham. Further results on generalized closed sets in topology- kyungpook Math .J .20(2) (1980) 169- 175.

[10] K. Kannan, On Levine 's Generalized closed sets. A survey, Research Journal of Applied Sciences, Engineering and Technology 4 (11) (2012) 1612-1615.

[11] N. Palaniappan and K. C. Rao: Regular generalized sets. Kyungpook math 33 (2) (1993) 211- 219.

[12] Sharmistha Bhattacharya. On Generalized Regular closed sets. Int. J. contemp. Math. Sciences, 6 (3) (2011) 145-452.

[13] A. E. Kornas and K. A. Arwini, R-Countability Axioms. An International Scientific Journal 149 (2020) 92-109.

[14] Majid Mirmiran, A Note On Extremally Disconnected Spaces. Research Open Journal Of Information Science, 1 (1) (2013) 01-03.

	مجـلة الـتربـوي Journal of Educational ISSN: 2011- 421X Arcif Q3	معامل التأثير العربي 1.5 العدد 19

الفهـــــــــــرس