



مجلة التربوي
Journal of Educational
ISSN: 2011- 421X
Arcif Q3

معامل التأثير العربي 1.5
العدد 19



مجلة التربوي

مجلة علمية محكمة تصدر عن كلية التربية

جامعة المرقب

العدد التاسع عشر
يوليو 2021م

هيئة تحرير
مجلة التربوي

- المجلة ترحب بما يرد عليها من أبحاث وعلى استعداد لنشرها بعد التحكيم .
 - المجلة تحترم كل الاحترام آراء المحكمين وتعمل بمقتضاها .
 - كافة الآراء والأفكار المنشورة تعبر عن آراء أصحابها ولا تتحمل المجلة تبعاتها .
 - يتحمل الباحث مسؤولية الأمانة العلمية وهو المسؤول عما ينشر له .
 - البحوث المقدمة للنشر لا ترد لأصحابها نشرت أو لم تنشر .
- (حقوق الطبع محفوظة للكلية)



ضوابط النشر:

- يشترط في البحوث العلمية المقدمة للنشر أن يراعى فيها ما يأتي :
- أصول البحث العلمي وقواعده .
 - ألا تكون المادة العلمية قد سبق نشرها أو كانت جزءا من رسالة علمية .
 - يرفق بالبحث تزكية لغوية وفق أنموذج معد .
 - تعدل البحوث المقبولة وتصحح وفق ما يراه المحكمون .
 - التزام الباحث بالضوابط التي وضعتها المجلة من عدد الصفحات ، ونوع الخط ورقمه ، والفترات الزمنية الممنوحة للتعديل ، وما يستجد من ضوابط تضعها المجلة مستقبلا .

تنبيهات :

- للمجلة الحق في تعديل البحث أو طلب تعديله أو رفضه .
- يخضع البحث في النشر لأولويات المجلة وسياساتها .
- البحوث المنشورة تعبر عن وجهة نظر أصحابها ، ولا تعبر عن وجهة نظر المجلة .

Information for authors

- 1- Authors of the articles being accepted are required to respect the regulations and the rules of the scientific research.
- 2- The research articles or manuscripts should be original and have not been published previously. Materials that are currently being considered by another journal or is a part of scientific dissertation are requested not to be submitted.
- 3- The research articles should be approved by a linguistic reviewer.
- 4- All research articles in the journal undergo rigorous peer review based on initial editor screening.
- 5- All authors are requested to follow the regulations of publication in the template paper prepared by the editorial board of the journal.

Attention

- 1- The editor reserves the right to make any necessary changes in the papers, or request the author to do so, or reject the paper submitted.
- 2- The research articles undergo to the policy of the editorial board regarding the priority of publication.
- 3- The published articles represent only the authors' viewpoints.





λ -Generalizations And g- Generalizations

Khadiga Ali Arwini^{1,*}, Entisar Othman Laghah²

¹Mathematics Department, Tripoli University, Tripoli-Libya

²Mathematics Department, Zawia University, Zawia-Libya
Kalrawini@yahoo.com*

ABSTRACT

In this paper, we use the concept of λ -closed set and g-closed set to define two classes of generalized regular closed sets; namely λ -generalizations and g-generalizations. The class of λ -generalizations includes: r λ -closed set, r* λ -closed set and r λ^* -closed set, while the second class of generalizations includes: g.r-closed set, g.r-closed set and r.g*-closed set. We investigate the characterizations of these generalizations, moreover, we illustrate the implications of these classes among themselves and with the known sets, and finally we study their behavior in regular spaces and in extremely disconnected spaces.

Keywords: Topological space and generalizations, regular space, extremely disconnected space.

AMS Subject Classification (2000): 54A05, 54D10, 54G05.

1. INTRODUCTION

The concept of regular closed sets was introduced by Stone in 1937 [1], where a subset in a topological space is called regular closed (briefly r-closed) if it equals to the closure of its interior, Stone studied this class of sets, and showed that r-closed set is stronger than closed set. The family of r-closed sets has some applications in the semiregularization space [1,2], also in a generalization for algebraic openings and closings in a complete lattice [3].

Many studies in the literature have been made on defining different generalizations of closed sets as; v-sets, g-closed sets, λ -closed sets, α -closed sets, semi-closed sets, preclosed sets, b-closed sets, etc., where these notions were defined using the closure and the interior operations. The concept of these generalizations play a significant role in general topology, and used to derive several forms of higher and lower separation axioms and compactness.

Maki [4] introduced the notion of Λ -sets in topological spaces, where a Λ -set is a set that equals to its kernel, i.e. to the intersection of all open supersets of the set, then in 2021, Almarghani and Arwini [5] introduced generalizations of regular closed sets, namely v_r -sets, g. v_r -sets, g*. v_r -set and g*. v_r -set, by considering the notion of the closure operator Λ_r -closure. Arenas et al. [6] introduced and investigated the notion of λ -closed sets by involving Λ -sets and closed sets, this enabled them to obtain new separation axioms by



utilizing the notion of λ -closure operator. The concept of generalized closed set (briefly g-closed) was due to Levine in 1970 [7], when he used this notation to define a space called $T_{1\frac{1}{2}}$ -space, and he showed that $T_{1\frac{1}{2}}$ is strictly between the spaces T_1 and T_0 [8,9,10]. In 1993, Palaniappan [11] introduced the concept of regular generalized closed sets (briefly r.g-closed) and he proved that this class of sets is weaker than the class of g-closed sets. Later on 2011 [12] Bhattacharya defined a new class of sets called generalized regular closed sets (briefly g.r-closed), when he studied the behavior relative to unions, intersections and subspaces; moreover, he proved that these class of sets are weakly ordered as; r-closed set, g.r-closed set, g-closed set then r.g-closed set.

The purpose of this article is to use the notions of λ -closed set and g-closed set to define two classes of generalized regular closed sets; namely λ -generalizations and g-generalizations, where λ -generalizations class consists the sets: r λ -closed sets, r* λ -closed sets and r λ^* -closed sets, while the second class of generalizations contains the sets: g.r-closed sets, g.r-closed sets and r.g*-closed sets, where g.r-closed sets and r.g-closed sets were due to Bhattacharya and Palaniappan, as we mentioned before. We discuss the properties of these generalizations, moreover, we illustrate the implications of these classes among themselves and with the known sets, and finally we investigate their behavior in regular spaces and in extremely disconnected spaces.

We divided our article into five main sections as; introduction, preliminaries, λ -generalizations, g-generalizations and finally conclusion.

2. PRELIMINARIES

In this section, we recall the definitions of regular-closed sets, v-sets, λ -closed sets and g-closed sets, with some of their properties that we need in the sequel.

Throughout this paper (X, τ) represented non-empty topological space, and will be replaced by X if there is no chance of confusion, no separation axioms assumed unless otherwise mentioned. If A is a subset of a space X , the notions \bar{A} and A° denote the closure and the interior of A ; respectively.

2.1. Regular Closed Sets

Definition 2.1.1. [1] A subset B of a space (X, τ) is called regular closed (briefly r-closed) if $B = \bar{B}^\circ$, while the set B is called δ -closed set if B is the intersection of r-closed sets. The family of all r-closed sets in (X, τ) is denoted by $RC(X, \tau)$.

Corollary 2.1.1. [1]

- 1- Every r-closed set is δ -closed set.
- 2- Every δ -closed set is closed set.
- 3- Intersection of r-closed sets is not necessarily r-closed.
- 4- Finite union of r-closed sets is r-closed.

Definition 2.1.2. [13] Let A be a subset of X then, the r-closure of A is defined as the intersection of all r-closed sets containing A , and is denoted \bar{A}^r .



Proposition 2.1.1. [13] Let X be a space and $A, B \subseteq X$, then:

- 1- \overline{A}^r is δ -closed set but not r -closed set in general.
- 2- $A \subseteq \overline{A} \subseteq \overline{A}^r$.
- 3- If A is r -closed then $A = \overline{A}^r$.
- 4- A is δ -closed if and only if $A = \overline{A}^r$.

Definition 2.1.3. [13] A space X is called regular-space if for any closed set F and $x \notin F$ there exist disjoint open sets U and V such that $x \in U$ and $F \subseteq V$.

Definition 2.1.4. [14] A space X is called extremely disconnected (briefly e.d) if, the closure of every open set in x is also open.

Proposition 2.1.2. [14] In extremely disconnected space (X, τ) ; we have:

- 1- Any r -closed set is clopen.
- 2- Any r -open set is clopen.
- 3- $RO(X, \tau) = RC(X, \tau) = \tau \cap \mathcal{F}$, where $RO(X, \tau)$ is the family of all r -open sets in X , and \mathcal{F} is the collection of all closed sets in X .

2. 2. Δ -Sets and Regular Δ -Sets

Definition 2.2.1. [4] Let B be a subset of a topological space (X, τ) , then:

- 1- $B^v = \cup \{F: F \subseteq B, F \text{ is closed}\}$.
- 2- $B^\Delta = \cap \{U: B \subseteq U, U \text{ is open}\}$.
- 3- $B^{v_r} = \cup \{N: N \subseteq B, N \text{ is } r\text{-closed}\}$.
- 4- $B^{\Delta_r} = \cap \{W: B \subseteq W, W \text{ is } r\text{-open}\}$.

Definition 2.2.2. [4] A subset B of a topological space (X, τ) is called:

- 1- v -set if $B^v = B$.
- 2- Δ -set if $B = B^\Delta$.
- 3- v_r -set if $B^{v_r} = B$.
- 4- Δ_r -set if $B = B^{\Delta_r}$.

Theorem 2.2.1. [5] Let A and B be subsets of a topological space (X, τ) , then the following properties are hold:

- 1- $B^{v_r} \subseteq B^v \subseteq B \subseteq B^\Delta \subseteq B^{\Delta_r}$.
- 2- If $A \subseteq B$ then $A^{v_r} \subseteq B^{v_r}$ and $A^{\Delta_r} \subseteq B^{\Delta_r}$.
- 3- $(B^{v_r})^{v_r} = B^{v_r}$.
- 4- $(B^{\Delta_r})^{\Delta_r} = B^{\Delta_r}$.
- 5- $(B^{v_r})^c = (B^c)^{\Delta_r}$.
- 6- $(B^{\Delta_r})^c = (B^c)^{v_r}$.



Theorem 2.2.2. [5] In a topological space (X, τ) the following hold:

- 1- Every r -closed is v_r -set.
- 2- Every v_r -set is v -set.
- 3- Every r -open is Λ_r -set.
- 4- Every Λ_r - set is Λ -set.
- 5- B is v_r -set iff B^c is Λ_r -set.

Diagram 1, shows the implications between the generalizations:

$$\begin{array}{ccc} r\text{-closed} \Rightarrow v_r\text{-set} & & r\text{-open} \Rightarrow \Lambda_r\text{-set} \\ \Downarrow & \Downarrow & \text{and} & \Downarrow & \Downarrow \\ \text{closed} \Rightarrow v\text{-set} & & & \text{open} \Rightarrow \Lambda\text{-set} \end{array}$$

Diagram 1. Generalizations of regular closed sets and regular open sets.

Theorem 2.2.3. [5] In e.d space (X, τ) , if $A \subseteq X$ then $A^{\wedge r} = \overline{A}^r$.

Theorem 2.2.4. [5] In regular space (X, τ) , if $A \subseteq X$, then $\overline{A}^r = \overline{A}$.

Corollary 2.2.1. [5] In regular e.d space X , if $A \subseteq X$, then $A^{\wedge r} = \overline{A}^r = \overline{A}$.

2. 3. λ -Closed Sets

Definition 2.3.1. [6] A subset A of a topological space (X, τ) is called λ -closed if $A = L \cap F$, where L is Λ -set and F is closed set.

Corollary 2.3.1. [6]

- 1- Every closed set is λ -closed
- 2- Every Λ -set is λ -closed.

Theorem 2.3.1. [6] For a subset A of a topological space (X, τ) the following statements are equivalent:

- 1- A is λ -closed.
- 2- $A = A^{\wedge} \cap \overline{A}$.

2. 4. g -Closed Sets

Definition 2.4.1. [10,11,12] A subset A of a topological space (X, τ) is said to be :

- 1- Generalized closed (briefly g -closed) if $\overline{A} \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- 2- Regular generalized closed (briefly $r.g$ -closed) if $\overline{A} \subseteq W$ whenever $A \subseteq W$ and W is r -open .



3- Generalized regular closed (briefly g.r-closed) if $\overline{A}^r \subseteq U$ whenever $A \subseteq U$ and U is open in X .

Corollary 2.4.1. [10]

- 1- Any closed set is g-closed.
- 2- Union of g-closed sets is g-closed.
- 3- Finite Intersection of g-closed sets is not necessarily g-closed.

Corollary 2.4.2. [10] If X is a topological space and x is a point in X such that $\{x\}$ is not closed, then $\{x\}^c$ is a g-closed set.

Theorem 2.4.1. [10] For a subset A of a topological space (X, τ) the following statements are equivalent:

- 1- A is closed.
- 2- A is g-closed and λ -closed.

3. λ -GENELAIZATIONS

In this section, we define a new class of generalizations by involving Δ_r -sets and r -closed sets; namely λ -generalizations that contains the sets: r λ -closed set, r^* λ -closed set and r λ^* -closed set. We prove that these sets are weakly ordered as: r -closed set, r λ -closed set, r λ^* -closed set, λ -closed set. In addition, we investigate the properties of this class of generalizations in regular spaces and in e.d spaces.

Definition 3.1. A subset A of a topological space (X, τ) is called:

- 1- r λ -closed if $A=L \cap F$, where L is Δ_r -set and F is r -closed set.
- 2- r λ^* -closed if $A=L \cap F$, where L is Δ -set and F is r -closed set.
- 3- r^* λ -closed if $A=L \cap F$, where L is Δ_r -set and F is closed set.

Theorem 3.1. In a topological space (X, τ) , we have:

- 1- Any r -closed set in X is r λ -closed and r λ^* -closed.
- 2- Any closed set in X is r^* λ -closed and λ -closed.
- 3- Every Δ_r -set is r λ -closed set and r^* λ -closed.
- 4- Every Δ -set is r λ^* -closed set and λ -closed.

Corollary 3.1.

- 1- Every r λ -closed set is r λ^* -closed and r^* λ -closed.
- 2- Every r λ^* -closed is λ -closed.
- 3- Every r^* λ -closed is λ -closed.

Proof:

- 1- Direct since every r -closed is closed and Δ_r -set is Δ -set.
- 2- Direct since every r -closed is closed
- 3- Direct since every Δ_r -set is Δ -set.



Example 3.1. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\}$, then $RC(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{a, d\}\}$. If $A = \{a, b, c\}$, $B = \{b, c\}$ and $C = \{d\}$, then:

- 3- A is $r\lambda^*$ -closed but not $r\lambda$ -closed; also A is λ -closed but not $r^*\lambda$ -closed.
- 4- B is $r\lambda$ -closed but not r -closed; also B is $r^*\lambda$ -closed but not closed.
- 5- C is λ -closed but not $r\lambda^*$ -closed; also C is $r^*\lambda$ -closed but not $r\lambda$ -closed.

Theorem 3.2. A subset A of a topological space (X, τ) is $r\lambda$ -closed iff $A = A^{\Delta_r} \cap \bar{A}^r$.

Proof: \Rightarrow Suppose A is $r\lambda$ -closed then $A = B \cap F$, where B is Δ_r -set and F is r -closed set, so $A \subseteq \bar{A}^r$ and $A \subseteq A^{\Delta_r}$, thus $A \subseteq A^{\Delta_r} \cap \bar{A}^r \dots (1)$, $A = B \cap F \subseteq F$ then $\bar{A}^r \subseteq F$ (F is r -closed), $A = B \cap F \subseteq B$ then $A^{\Delta_r} \subseteq B$ (B is Δ_r -set) and $A^{\Delta_r} \cap \bar{A}^r \subseteq B \cap F$, i.e. $A^{\Delta_r} \cap \bar{A}^r \subseteq A \dots (2)$. From (1) and (2) we get $A = A^{\Delta_r} \cap \bar{A}^r$.

\Leftarrow Suppose $A = A^{\Delta_r} \cap \bar{A}^r$, since A^{Δ_r} is Δ_r -set and \bar{A}^r is r -closed then A is $r\lambda$ -closed.

Theorem 3.3. A subset A of a topological space (X, τ) is $r\lambda^*$ -closed iff $A = A^\Delta \cap \bar{A}^r$.

Proof: \Rightarrow Suppose A is $r\lambda^*$ -closed, so $A = B \cap F$ where B is Δ -set and F is r -closed set, and since $A \subseteq \bar{A}^r$ and $A \subseteq A^\Delta$ we have $A \subseteq A^\Delta \cap \bar{A}^r \dots (1)$, now since $A = B \cap F \subseteq F$, then $\bar{A}^r \subseteq F$ (F is r -closed), $A = B \cap F \subseteq B$ then $A^\Delta \subseteq B$ (B is Δ -set), we obtain $A^\Delta \cap \bar{A}^r \subseteq B \cap F$, i.e. $A^\Delta \cap \bar{A}^r \subseteq A \dots (2)$. From (1) and (2) we get $A = A^\Delta \cap \bar{A}^r$.

\Leftarrow Suppose $A = A^\Delta \cap \bar{A}^r$, since A^Δ is Δ -set and \bar{A}^r is r -closed then A is $r\lambda^*$ -closed.

Theorem 3.4. A subset A of a topological space (X, τ) is $r^*\lambda$ -closed iff $A = A^{\Delta_r} \cap \bar{A}$.

Proof: \Rightarrow Suppose A is $r^*\lambda$ -closed, so $A = T \cap C$, where T is Δ_r -set and C is closed set, since $A \subseteq \bar{A}$ and $A \subseteq A^{\Delta_r}$ then $A \subseteq A^{\Delta_r} \cap \bar{A} \dots (1)$, now $A = T \cap C \subseteq C$ then $\bar{A} \subseteq C$ (C is closed), $A = T \cap C \subseteq T$ so $A^{\Delta_r} \subseteq T$ (T is Δ_r -set), we have $A^{\Delta_r} \cap \bar{A} \subseteq T \cap C$, i.e. $A^{\Delta_r} \cap \bar{A} \subseteq A \dots (2)$. From (1) and (2) we get $A = A^{\Delta_r} \cap \bar{A}$.

\Leftarrow Suppose $A = A^{\Delta_r} \cap \bar{A}$, since A^{Δ_r} is Δ_r -set and \bar{A} is closed then A is $r^*\lambda$ -closed.

Theorem 3.5. In regular space (X, τ) if $A \subseteq X$ then:

- 1- A is $r\lambda^*$ -closed iff A is λ -closed.
- 2- A is $r\lambda$ -closed iff A is $r^*\lambda$ -closed.

Proof:

1- \Rightarrow Direct.

\Leftarrow If A is λ -closed then from theorems (2.3.1), (2.2.4) and (3.3) we obtain $A = A^\Delta \cap \bar{A} = A^\Delta \cap \bar{A}^r$, so A is $r\lambda^*$ -closed.

2- \Rightarrow Direct.

\Leftarrow If A is $r^*\lambda$ -closed then from theorems (3.4), (2.2.4) and (3.2) we obtain $A = A^{\Delta_r} \cap \bar{A} = A^{\Delta_r} \cap \bar{A}^r$, so A is $r\lambda$ -closed.



Theorem 3.6. In e.d space (X, τ) if $A \subseteq X$ then:

- 1- A is $r\lambda$ -closed iff $\overline{A}^r = A$.
- 2- A is $r\lambda^*$ -closed iff A is Λ -set.
- 3- A is $r^*\lambda$ -closed iff A is closed.

Proof:

- 1- \Rightarrow If A is $r\lambda$ -closed, then $A = A^{\Lambda r} \cap \overline{A}^r = \overline{A}^r \cap \overline{A}^r = \overline{A}^r$.
 \Leftarrow If $\overline{A}^r = A$, then $A^{\Lambda r} \cap \overline{A}^r = A^{\Lambda r} \cap A = A$, so A is $r\lambda$ -closed.
- 2- \Rightarrow If A is $r\lambda^*$ -closed, then $A = A^\Lambda \cap \overline{A}^r = A^\Lambda \cap A^{\Lambda r} = A^\Lambda$, so A is Λ -set.
 \Leftarrow If A is Λ -set, then $A^\Lambda \cap \overline{A}^r = A \cap \overline{A}^r = A$, so A is $r\lambda^*$ -closed.
- 3- \Rightarrow If A is $r^*\lambda$ -closed, then $A = A^{\Lambda r} \cap \overline{A}^r = \overline{A}^r \cap \overline{A}^r = \overline{A}^r$, so A is closed.
 \Leftarrow If A is closed, then $A^{\Lambda r} \cap \overline{A}^r = A^{\Lambda r} \cap A = A$, so A is $r^*\lambda$ -closed.

Corollary 3.2. In e.d regular space X , if $A \subseteq X$, then these statements are equivalent:

- 1- A is Λ -set.
- 2- A is λ -closed.
- 3- A is $r\lambda^*$ -closed.

Proof: Direct from the previous theorems.

Corollary 3.3. In e.d regular space X , if $A \subseteq X$, then these statements are equivalent:

- 1- A is $r\lambda$ -closed.
- 2- A is $r^*\lambda$ -closed.
- 3- A is closed.
- 4- $\overline{A}^r = A$.

Proof: Direct from theorems (3.5) and (3.6).

4. g-GENERALIZATIONS

In the present section, we define a new generalization of r -closed sets; namely $r.g^*$ -closed sets. We study their properties and illustrate the implication of this set with the known sets; as is g -closed set, $r.g$ -closed set and $g.r$ -closed set, then we investigate the behaviour of these generalizations in regular spaces and in e.d spaces.

Definition 4.1. A subset A of a space X is said to be regular generalized star closed (briefly $r.g^*$ -closed) if $\overline{A}^r \subseteq W$ whenever $A \subseteq W$ and W is r -open in X .

Corollary 4.1. In any a topological space (X, τ) these statements are hold:

- 1- Every r -closed set is $g.r$ -closed.
- 2- Every $g.r$ -closed set is $r.g^*$ -closed.
- 3- Every $r.g^*$ -closed set is $r.g$ -closed.
- 4- Every g -closed set is $r.g$ -closed.



Proof:

- 1- Direct since $\bar{A}^r = A$
- 2- Direct since every r-open set is open.
- 3- Direct since $\bar{A} \subseteq \bar{A}^r$ for any subset A in X.
- 4- Direct since every r-open set is open.

Examples 4.1. In the cofinite topology on \mathbb{R} , we have $RO(\mathbb{R}, \tau_c) = \{\mathbb{R}, \emptyset\}$. If A is any non-empty finite set then $\bar{A}^r = \mathbb{R}$ and the only r-open set W such that $A \subseteq W$ is $W = \mathbb{R}$, so A is r.g*-closed, but not g.r-closed since $\bar{A}^r = \mathbb{R}$ and $U = \{x\}^c$, where $x \notin A$ is an open set contains A, but $\bar{A}^r \not\subseteq U$.

Corollary 4.2. If X is a topological space and x is a point in X such that $\{x\}$ is not r-closed, then $\{x\}^c$ is r.g-closed set.

Theorem 4.1. A subset A in a topological space (X, τ) is g-closed iff $\bar{A} \subseteq A^\Delta$.

Proof: \Rightarrow Let A be a g-closed set in X, and let V be an open set such that $A \subseteq V$, then $A \subseteq A^\Delta \subseteq V$, since A is g-closed and V is open then $\bar{A} \subseteq V$, i.e. for any open set V such that $A \subseteq V$ we have $\bar{A} \subseteq V$, so $\bar{A} \subseteq \bigcap_{A \subseteq V} V$. Hence $\bar{A} \subseteq A^\Delta$.

\Leftarrow Suppose $\bar{A} \subseteq A^\Delta$ and V is an open set such that $A \subseteq V$, then $A \subseteq A^\Delta \subseteq V$, so $\bar{A} \subseteq A^\Delta \subseteq V$, i.e. A is g-closed set.

Theorem 4.2. A subset A in a topological space (X, τ) is g.r-closed iff $\bar{A}^r \subseteq A^\Delta$.

Proof: \Rightarrow For any open set U in X such that $A \subseteq U$ we have $A \subseteq A^\Delta \subseteq U$ and $\bar{A}^r \subseteq U$ since A is g.r-closed so $\bar{A}^r \subseteq \bigcap_{A \subseteq U} U$. U is an open set, then $\bar{A}^r \subseteq A^\Delta$.

\Leftarrow Suppose $\bar{A}^r \subseteq A^\Delta$, and U is an open set in X such that $A \subseteq U$, then $A \subseteq A^\Delta \subseteq U$ we have $\bar{A}^r \subseteq A^\Delta \subseteq U$ then $\bar{A}^r \subseteq U$. Hence A is g.r-closed.

Theorem 4.3. A subset A in a topological space (X, τ) is r.g*-closed iff $\bar{A}^r \subseteq A^{\Delta r}$.

Proof: \Rightarrow Suppose A is r.g*-closed set, then for any r-open set W in X such that $A \subseteq W$ we have $A \subseteq A^{\Delta r} \subseteq W$ since A is r.g*-closed, we get $\bar{A}^r \subseteq W$, then $\bar{A}^r \subseteq \bigcap_{A \subseteq W} W$, W is r-open so $\bar{A}^r \subseteq A^{\Delta r}$.

\Leftarrow Suppose $\bar{A}^r \subseteq A^{\Delta r}$ and W is r-open set such that $A \subseteq W$ then $A \subseteq A^{\Delta r} \subseteq W$ so $\bar{A}^r \subseteq A^{\Delta r} \subseteq W$, i.e. $\bar{A}^r \subseteq W$ hence A is r.g*-closed.

Corollary 4.3. For a subset A of a topological space (X, τ) , we have:

- 1- If A is r-closed set then A is r.g*-closed and r λ -closed.
- 2- If A is r.g*-closed and r λ -closed then A is δ -closed.



Proof:

- 1- Direct from theorem (3.1) and corollary (4.1).
- 2- Since A is $r.g^*$ -closed and $r\lambda$ -closed, we have $\bar{A}^r \subseteq A^{\wedge r}$ and $A = A^{\wedge r} \cap \bar{A}^r$, then $\bar{A}^r \subseteq A^{\wedge r} \cap \bar{A}^r$, so $\bar{A}^r \subseteq A$, i.e. $\bar{A}^r = A$. We get A is δ -closed.

Corollary 4.4. In e.d space (X, τ) , any subset of X is $r.g$ -closed and $r.g^*$ -closed.

Proof: Let $A \subseteq X$, and W is r -open set such that $A \subseteq W$, since any r -open set in e.d space is r -closed, then $\bar{A}^r \subseteq \bar{W}^r$ so $\bar{A}^r \subseteq W$ and we have $\bar{A} \subseteq \bar{A}^r \subseteq W$, thus A is $r.g$ -closed and $r.g^*$ -closed.

Corollary 4.5. In regular space (X, τ) , a subset A of X is $r.g$ -closed iff A is $r.g^*$ -closed.

Proof: \Leftarrow Direct since any $r.g^*$ -closed is $r.g$ -closed.

\Rightarrow Suppose A is $r.g$ -closed and $A \subseteq W$ when W is r -open set, then $\bar{A} \subseteq W$ since X is regular, we have $\bar{A}^r = \bar{A}$, so $\bar{A}^r \subseteq W$, hence A is $r.g^*$ -closed.

Corollary 4.6. In regular space (X, τ) , any subset A of X is g -closed iff A is $g.r$ -closed.

Proof: \Leftarrow Direct.

\Rightarrow Suppose A is g -closed, and $A \subseteq U$ where U is an open set, then $\bar{A} \subseteq U$, since X is regular space $\bar{A}^r = A$ so $\bar{A}^r \subseteq \bar{A} \subseteq U$, thus A is $g.r$ -closed.

CONCLUSION

In this paper, we introduce new classes of generalizations using the notions of λ -closed sets and g -closed sets; namely λ -generalizations and g -generalizations. The first class of generalizations includes; $r\lambda$ -closed set, $r^*\lambda$ -closed set and $r\lambda^*$ -closed set, while the class of g -generalizations includes; $g.r$ -closed set, $g.r$ -closed set and $r.g^*$ -closed set, where $g.r$ -closed and $g.r$ -closed were due to Bhattacharya and Palaniappan. The characterizations of these generalizations are studied, moreover, we illustrate the implications between these sets, and finally we study their behavior in regular spaces and in extremely disconnected spaces.

Here we summarize our results:

- The following diagrams show the implications between the new classes of generalizations:

$$\begin{array}{ccccc} r\text{-closed} & \Rightarrow & r\lambda\text{-closed} & \Rightarrow & r\lambda^*\text{-closed} \\ \Downarrow & & \Downarrow & & \Downarrow \\ \text{Closed} & \Rightarrow & r^*\lambda\text{-closed} & \Rightarrow & \lambda\text{-closed} \end{array}$$



Diagram 2. Implication between the class of λ -generalization sets.

$$\begin{array}{ccccc} r\text{-closed} & \Rightarrow & g.r\text{-closed} & \Rightarrow & r.g^*\text{-closed} \\ \Downarrow & & \Downarrow & & \Downarrow \\ \text{Closed} & \Rightarrow & g\text{-closed} & \Rightarrow & r.g\text{-closed} \end{array}$$

Diagram 3. Implication between the class of g -generalization sets.

- In any space X , if a singleton $\{x\}$ is not r -closed then $\{x\}^c$ is $r.g$ -closed set.
- In any space X , if $A \subseteq X$ then:
 - A is g -closed iff $\bar{A} \subseteq A^\wedge$.
 - A is $g.r$ -closed iff $\bar{A}^r \subseteq A^\wedge$.
 - A is $r.g^*$ -closed iff $\bar{A}^r \subseteq A^{\wedge r}$.
 - If A is $r.g^*$ -closed and r λ -closed then A is δ -closed.
- In regular space X , if $A \subseteq X$ then:
 - $\bar{A}^r = \bar{A}$.
 - A is λ -closed set iff A is $r\lambda^*$ -closed.
 - A is $r\lambda$ -closed set iff A is $r^*\lambda$ -closed.
 - A is g -closed-set iff A is $g.r$ -closed.
 - A is $r.g$ -closed-set iff A is $r.g^*$ -closed.
- In e.d space X , if $A \subseteq X$ then:
 - $\bar{A}^r = A^{\wedge r}$.
 - A is closed iff A is $r^*\lambda$ -closed.
 - A is Λ -set iff A is $r\lambda^*$ -closed.
 - $\bar{A}^r = A$ iff A is $r\lambda$ -closed.
 - Any subset of X is $r.g$ -closed and $r.g^*$ -closed.
- In regular e.d space X , if $A \subseteq X$ then these statements are equivalent:
 - A is Λ -set.
 - A is λ -closed.
 - A is $r\lambda^*$ -closed.
- In regular e.d space X , if $A \subseteq X$ then these statements are equivalent:
 - A is $r\lambda$ -closed.
 - A is $r^*\lambda$ -closed.
 - A is closed.
 - $\bar{A}^r = A$.

References

- [1] M. H. Stone, Applications Of The Theory Of Boolean Rings To General Topology. Trans. Am. Math. Soc., 41(1937) 375-481.
- [2] Hisham Mahdi, Fadwa Nasser. On Minimal and Maximal Regular Open Sets, Mathematics and Statistics 5 (2) (2017) 78-83.



- [3] C. Ronse. Regular Open or Closed Sets, Philips Research Laboratory Brussels, A v. E van Beclaere 2 B (1990) 1170.
- [4] H. Maki, Generalized Λ -Sets and The Associated Closure Operator. The Special Issue in Commemoration of Prof. Kazusada IKEDA's Retirement, 1 (1986) 139–146.
- [5] Mabrouka Almarghani and K. A. Arwini, Generalizations of Regular Closed Set. An International Scientific Journal 152 (2021) 55-68
- [6] F.G. Arenas, J. Dontchev and M.L. Puertas, Unification approach to the separation axioms between T_0 and completely hausdorff. Acta Math. Hungar (2000).
- [7] N. Levine, Generalized Closed Sets In Topology. Rend. Circ. Mat. Palermo, 19 (2) (1970) 89-96.
- [8] N. Levine A decomposition of continuity in topological spaces, Amer. Math. Monthly, 68 (1961), 44–46.
- [9] Levine, N and W. Dunham. Further results on generalized closed sets in topology-kyungpook Math .J .20(2) (1980) 169- 175.
- [10] K. Kannan, On Levine 's Generalized closed sets. A survey, Research Journal of Applied Sciences, Engineering and Technology 4 (11) (2012) 1612-1615.
- [11] N. Palaniappan and K. C. Rao: Regular generalized sets. Kyungpook math 33 (2) (1993) 211- 219.
- [12] Sharmistha Bhattacharya. On Generalized Regular closed sets. Int. J. contemp. Math. Sciences, 6 (3) (2011) 145-452.
- [13] A. E. Kornas and K. A. Arwini, R-Countability Axioms. An International Scientific Journal 149 (2020) 92-109.
- [14] Majid Mirmiran, A Note On Extremally Disconnected Spaces. Research Open Journal Of Information Science, 1 (1) (2013) 01-03.



الفهرس

الصفحة	اسم الباحث	عنوان البحث	ر.ت
1-23	يونس يوسف أبونايجي	وضع الضاهر موضع الضمير ودلالته على المعنى عند المفسرين	1
24-51	محمد خليفة صالح خليفة محمود الجداوي	دراسة استقصائية حول مساهمة تقنية المعلومات والاتصالات في نشر ثقافة الشفافية ومحاربة الفساد	2
52-70	Ebtisam Ali Haribash	An Interactive GUESS Method for Solving Nonlinear Constrained Multi-Objective Optimization Problem	3
71-105	احمد علي الهادي الحويج احمد محمد سليم معوال	العوامل الخمسة الكبرى للشخصية وعلاقتها بالذكاء الوجداني لدى طلبة مرحلة التعليم الثانوي	4
106-135	محمد عبد السلام دخيل	في المجتمع الليبي التحضر وانعكاساته على الحياة الاجتماعية "دراسة ميدانية في مدينة الخمس"	5
136-158	سالم فرج زويبيك	الاستعارة التهكمية في القرآن الكريم	6
159-173	أسماء جمعة القلعي	دور الرياضات العملية الصوفية في تهذيب السلوك	7
174-183	S. M. Amsheri N. A. Abouthferah	On Coefficient Bounds for Certain Classes of Analytic Functions	8
184-191	N. S. Abdanabi	Fibrewise Separation axioms in Fibrewise Topological Group	9
192-211	Samah Taleb Mohammed	Investigating Writing Errors Made by Third Year Students at the Faculty of Education El-Mergib University	10
212-221	Omar Ali Aleyan Eissa Husen Muftah AL remali	SOLVE NONLINEAR HEAT EQUATION BY ADOMIAN DECOMPOSITION METHOD [ADM]	11
222-233	حسن احمد قرقد عبدالباسط محمد قريصة مصطفى الطويل	قياس تركيز بعض العناصر الثقيلة في المياه الجوفية لمدينة مصراته	12
234-244	ربيعة عبد الله الشبير عائشة أحمد عامر عبير مصطفى الهصيك	تعادم الدوال الكروية المناظرة لقيم ذاتية على سطح الكرة	13
245-255	Khadiga Ali Arwini Entisar Othman Laghah	λ -Generalizations And g - Generalizations	14



256-284	خيري عبدالسلام حسين كليب عبدالسلام بشير اشتيوي بشير ناصر مختار كصارة	Impact of Information Technology on Supply Chain management	15
285-294	Salem H. Almadhun, Salem M. Aldeep, Aimen M. Rmis, Khairia Abdulsalam Amer	Examination of 4G (LTE) Wireless Network	16
295-317	نور الدين سالم فريوع	التجربة الجمالية لدى موريس ميرلوبوتي	17
318-326	ليلى منصور عطية الغويج هدى على التقبي	Effect cinnamon plant on liver of rats treated with trichloroethylene	18
327-338	Fuzi Mohamed Fartas Naser Ramdan Amaizah Ramdan Ali Aldomani Husamaldin Abdualmawla Gahit	Qualitative Analysis of Aliphatic Organic Compounds in Atmospheric Particulates and their Possible Sources using Gas Chromatography Mass Spectrometry	19
339-346	E. G. Sabra A. H. EL- Rifae	Parametric Tension on the Differential Equation	20
347-353	Amna Mohamed Abdelgader Ahmed	Totally Semi-open Functions in Topological Spaces	21
354-376	زينب إمام أبو راس حواء بشير بالنور	كتاب الخصائص لابن جني دراسة بعض مواضع الحذف من ت"392" المسمى: باب في شجاعة العربية	22
377-386	لطيفة محمد الدالي	Least-Squares Line	23
387-397	نادية محمد الدالي ايمان احمد اخميرة	THEORETICAL RESEARCH ON AI TECHNOLOGIES FOR LEARNING SYSEM	24
398-409	Ibrahim A. Saleh Tarek M. Fayez Mustafah M. A. Ahmad	Influence of annealing and Hydrogen content on structural and optoelectronic properties of Nano-multilayers of a-Si:H/a-Ge: H used in Solar Cells	25
410-421	أسماء محمد الحبشي	The learners' preferences of oral corrective feedback techniques	26
422-459	أمينة محمد العكاشي ربيعة عثمان عبد الجليل عفاف محمد بالحاج فتحية علي جعفر	التقدير الإيجابي المسبق لفاعلية الذات ودوره في التغلب على مصادر الضغوط النفسية " دراسة تحليلية "	27



460-481	Aisha Mohammed Ageal Najat Mohammed Jaber	English Pronunciation problems Encountered by Libyan University Students at Faculty of Education, Elmergib University	28
482-499	الحسين سليم محسن	The Morphological Analysis of the Quranic Texts	29
500-507	Ghada Al-Hussayn Mohsen	Cultural Content in Foreign Language Learning and Teaching	30
508-523	HASSAN M. ALI Mostafa M Ali	The relationship between <i>slyA</i> DNA binding transcriptional activator gene and <i>Escherichia coli</i> fimbriae and related with biofilm formation	31
524-533	Musbah A. M. F. Abduljalil	Molecular fossil characteristics of crude oils from Libyan oilfields in the Zalla Trough	32
534-542	سعدون شهبوب محمد	تلوث المياه الجوفية بالنترات بمنطقة كعام، شمال غرب ليبيا	33
543-552	Naima M. Alsharif Mahmoud M. Buazzi	Analysis of Genetic Diversity of <i>Escherichia Coli</i> Isolates Using RAPD PCR Technique	34
553-560	Hisham mohammed alnaib alshareef aisha mohammed elfagaeh aisha omran alghawash abdualaziz ibrahim lawej safa albashir hussain kaka	The Emergence of Virtual Learning in Libya during Coronavirus Pandemic	35
561-574	Abdualaziz Ibrahim Lawej Rabea Mansur Milad Mohamed Abduljalil Aghnayah Hamza Aabeed Khalafllaa ³	ATTITUDES OF TEACHERS AND STUDENTS TOWARDS USING MOTHER TONGUE IN EFL CLASSROOMS IN SIRTE	36
575-592	صالحة التومي الدروقي أمال محمد سالم أبوسته	دافع الانجاز وعلاقته بالرضا الوظيفي لدى معلمي مرحلة التعليم الأساسي "ببلدية ترهونة"	37
593-609	آمنة سالم عبد القادر قدورة نجية علي جبريل انبية	الإرشاد النفسي ودوره في مواجهة بعض المشكلات الأخرية الراهنة	38
610-629	Hanan B. Abousittash, Z. M. H. Kheiralla Betiha M.A.	Effect Mesoporous silica silver nanoparticles on antibacterial agent Gram- negative <i>Pseudomonas</i> <i>aeruginosa</i> and Gram-positive <i>Staphylococcus</i> <i>aureus</i>	39
630-652	حنان عمر بشير الرمالي	برنامج التربية العملية وتطويره	40
653-672	Abdualla Mohamed Dhaw	Towards Teaching CAT tools in Libyan Universities	41



673-700	عثمان علي أميمن سليمة رمضان الكوت زهرة عثمان البرق	سبل إعادة أعمار وتأهيل سكان المدن المدمرة بالحرب ومعوقات المصالحة الوطنية في المجتمع الليبي: مقارنة نفس-اجتماعية	42
701-711	Abdulrhman Mohamed Egnebr	Comparison of Different Indicators for Groundwater Contamination by Seawater Intrusion on the Khoms city, Libya	43
712-734	Elhadi A. A. Maree Abdualah Ibrahim Sultan Khaled A. Alurffi	Hilbert Space and Applications	44
735-759	معتوق علي عون عمار محمد الزليطني عرفات المهدي قرينات	الموارد الطبيعية اللازمة لتحقيق التنمية الاقتصادية بشمال غرب ليبيا وسبل تحقيق الاستدامة	45
760-787	سهام رجب العطوي هدى المبروك موسى	الخلج وعلاقته بمفهوم الذات لدى تلاميذ الشق الثاني بمرحلة التعليم الاساسي بمنطقة جنزور	46
788-820	هنية عبدالسلام بالوص زهرة المهدي أبو راس	الصلابة النفسية ودورها الوقائي في مواجهة الضغوط النفسية	47
821-847	عبد الحميد مفتاح أبو النور محي الدين علي المبروك	ودوره في الحد من التمر التوجيه التربوي والإرشاد النفسي المدرسي	48
848	الفهرس		52