Nonlinear Structural Dynamic Response of Multi-Story Buildings Under Seismic Loading

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ABSTRACT

A study of earthquakes in the world is performed, and also a study of methods of dynamic analysis suitable for use with linear and nonlinear systems is made with a stress on the nonlinear response of buildings due to moderate or high seismic loading.
The response of a building to a seismic load severe enough may induce inelastic deformations and the building behavior is expected to be nonlinear. Consequently, it is necessary to develop a method of analysis suitable for use with nonlinear systems. A step-by-step method is well suited to the analysis of nonlinear systems rather than using the method of superposition. The total structural response is due to each response contribution within the step. The Wilson-θ method is the step- by- step unconditionally stable method which is used for this aspect of nonlinear dynamic analysis and it is introduced in the present work.
Keywords— Large earthquake; nonlinear system; unconditionally stable.

1 Introduction

About 50,000 earthquakes occur annually over the entire earth, approximately 100 are of sufficient size to produce substantial damage if their centers are near areas of habitation. When a large earthquake occurs it may induce inelastic deformations and the building behavior is expected to be nonlinear. It is important to note that linear response analysis, whether formulated in the time domain or in the frequency domain involves the evaluation of many independent response contributions that are combined to obtain the total response. Hence, superposition technique is employed.
These methods employed only superposition method for linear systems. Neither one of them is suited for use in the analysis of nonlinear systems.
The step-by-step method is only conditionally stable and for numerical stability of the solution may require such an extremely small time step as to make the method impractical if not impossible. The Wilson-θ method serves to assure the numerical stability of the solution process regardless of the magnitude selected for the time step. For this reason, such a method is *unconditionally* stable. [1] [4] [6]

2 Linear and Nonlinear Behavior of Materials

Linear behavior is always elastic while nonlinear material behavior may be elastic or inelastic. If the spring is elastic then its force-displacement line or curve will follow the same curve, in unloading case as shown in Figures 1 and 2.

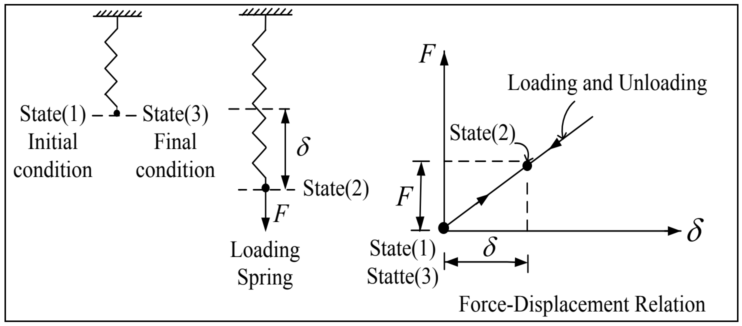

Figure 1: Linear spring: Elastic behavior

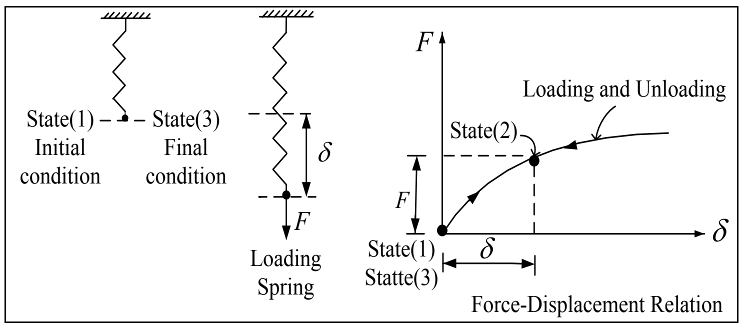

Figure 2: Nonlinear spring: Elastic behavior

If the material is inelastic and nonlinear as shown in Figure 3. Then in case of unloading, it does not follow the loading curve but follows a line parallel to the initial tangent of the loading curve. Permanent deformation δ^* will result.

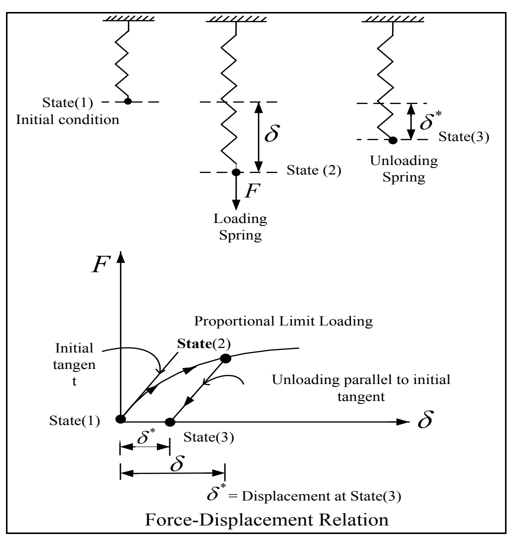

Figure 3: Nonlinear spring: Inelastic behavior

3 Linear and Nonlinear Behavior of Structures

Linear and nonlinear behavior of springs, under loading conditions are shown in the following two examples:

A. Linear Springs

Linear spring behaves linearly under loading conditions. Loading sequence is not important in case of linear material as shown in Figure 4. The resulting displacement is the same.

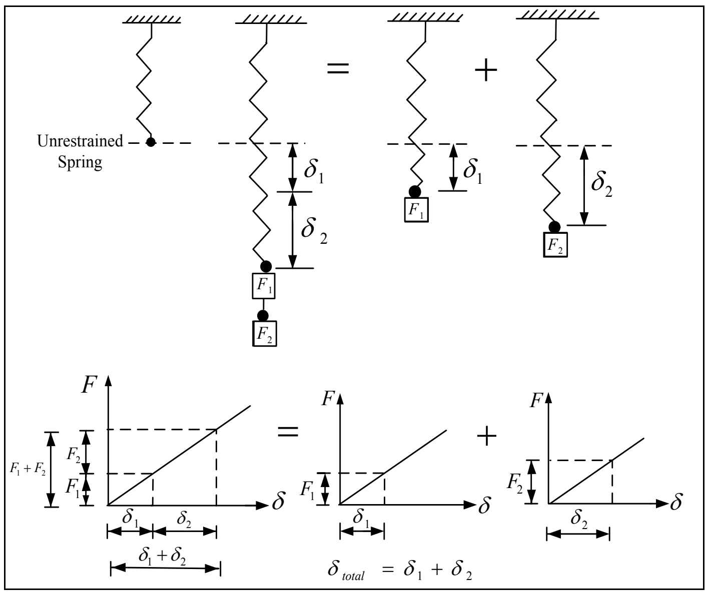

Figure 4: Superposition method applies to linear systems

Loading history is not relevant to the effect of a particular force. For example, force F_2 will always cause the same displacement δ_2 , no matter what loads are placed on the spring before it is applied.

B. Nonlinear Springs

Nonlinear spring is shown in Figure 5 the addition of loading does not result in the same effect in terms of displacements. Here, the effect of the forces F_1 and F_2 when applied together is not the same as the sum of the effects of these forces when each of them is applied separately.

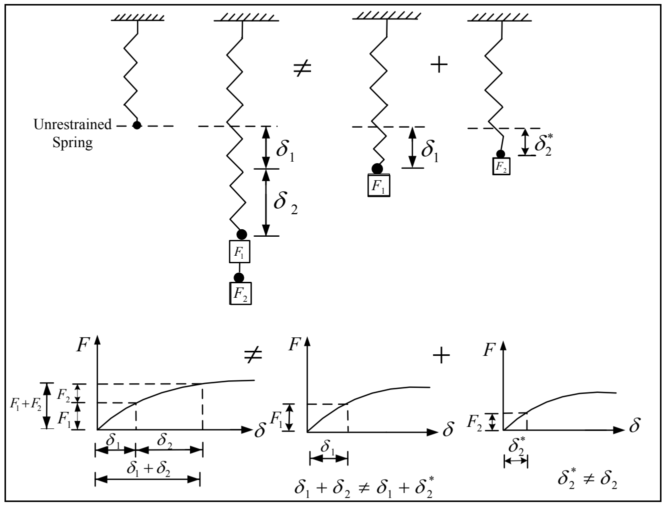

Figure 5: Superposition method does not apply for nonlinear systems

Therefore, superposition is not applicable to nonlinear springs. Loading history is relevant to the effect of a particular loading. The displacement due to a given load depends on the total force that is presently acting on a structure system or spring. Hence, superposition does not apply to nonlinear springs, which have the nonlinear force-displacement relationship.

There are many important classes of structural dynamic problems which cannot be assumed to be linear. The response of a building to a seismic load severe enough to induce inelastic deformations makes the building behavior nonlinear.

The step-by-step linear acceleration method is well suited to the analysis of nonlinear systems. The response for each time step is an independent analysis problem, and there is no need to combine response contributions within the step. This method is an explicit solution which is only *conditionally stable* and for numerical stability of the solution may require such an extremely small time step. The modification introduced to the method by Wilson serves to assure the numerical stability of the solution process regardless of the magnitude selected for the time step. For this reason, such a method is *unconditionally stable* and is suitable for this aspect for nonlinear dynamic analysis. [1] [4] [6]

4 Wilson-θ Method

The basic assumption of the Wilson-θ method is that the acceleration varies linearly over the time interval from t to $t+\theta\Delta t$ where $\theta \geq 1.0$. The value of the factor θ is determined to obtain optimum stability of the numerical process and accuracy of the solution. It has been shown by Wilson that for $\theta \geq 1.38$, the method becomes unconditionally stable. [6] [7]

Using the difference between dynamic equilibrium conditions defined at time t_i and $t_i+\tau$, where $\tau = \theta\Delta t$, we obtain the incremental equations

$$M\hat{\Delta}\ddot{u}_i + C(\dot{u})\hat{\Delta}\dot{u}_i + K(u)\hat{\Delta}u_i = \hat{\Delta}P_i \tag{1}$$

In which the circumflex over Δ indicates that the increments are associated with the extended step $\tau = \theta\Delta t$. Thus

$$\hat{\Delta}u_i = u(t_i + \tau) - u(t_i) \quad (2)$$

$$\hat{\Delta}\dot{u}_i = \dot{u}(t_i + \tau) - \dot{u}(t_i) \quad (3)$$

$$\hat{\Delta}\ddot{u}_i = \ddot{u}(t_i + \tau) - \ddot{u}(t_i) \quad (4)$$

$$\hat{\Delta}P_i = P(t_i + \tau) - P(t_i) \quad (5)$$

The stiffness coefficient and damping coefficient are obtained for each time step

$$K_{ij} = dF_{Si} / du_j \quad (6)$$

$$C_{ij} = dF_{Di} / d\dot{u}_j \quad (7)$$

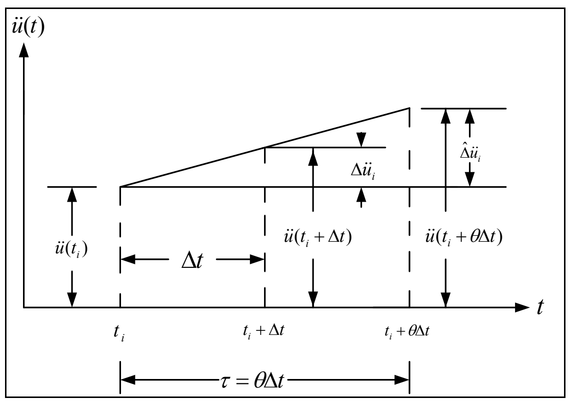

Figure 6: Linear acceleration assumption in the extended time interval.

From Figure 6, we can write the linear expression for the acceleration during the extended time step as

$$\ddot{u}(t) = \ddot{u}_i + \frac{\hat{\Delta}\ddot{u}_i}{\tau}(t - t_i) \quad (8)$$

In which $\hat{\Delta}\ddot{u}_i$ is given by (4), integrating (8) twice yields

$$\dot{u}(t) = \dot{u}_i + \ddot{u}_i(t - t_i) + \frac{1}{2}\frac{\hat{\Delta}\ddot{u}_i}{\tau}(t - t_i)^2 \quad (9)$$

$$u(t) = u_i + \dot{u}_i(t - t_i) + \frac{1}{2}\ddot{u}_i(t - t_i)^2 + \frac{1}{6}\frac{\hat{\Delta}\ddot{u}_i}{\tau}(t - t_i)^3 \quad (10)$$

Evaluation of (9) and (10) at the end of the extended interval $t = t_i + \tau$ gives

$$\hat{\Delta}\dot{u}_i = \ddot{u}_i\tau + \frac{1}{2}\hat{\Delta}\ddot{u}_i\tau \quad (11)$$

$$\hat{\Delta}u_i = \dot{u}_i\tau + \frac{1}{2}\ddot{u}_i\tau^2 + \frac{1}{6}\hat{\Delta}\ddot{u}_i\tau^2 \quad (12)$$

Equation (12) is solved for the incremental acceleration $\hat{\Delta}\ddot{u}_i$ and substituted in (11), we obtain

$$\hat{\Delta}\ddot{u}_i = \frac{6}{\tau^2}\hat{\Delta}u_i - \frac{6}{\tau}\dot{u}_i - 3\ddot{u}_i \tag{13}$$

$$\hat{\Delta}\dot{u}_i = \frac{3}{\tau}\hat{\Delta}u_i - 3\dot{u}_i - \frac{\tau}{2}\ddot{u}_i \tag{14}$$

Substituting (13) and (14) into (1), results in an equation for the incremental displacement $\hat{\Delta}u_i$ which may be conveniently written as

$$\overline{K}_i\hat{\Delta}u_i = \hat{\Delta}\overline{P}_i \tag{15}$$

where

$$\overline{K}_i = K_i + \frac{6}{\tau^2}M + \frac{3}{\tau}C_i \tag{16}$$

$$\hat{\Delta}\overline{P}_i = \hat{\Delta}P_i + M(\frac{6}{\tau}\dot{u}_i + 3\ddot{u}_i) + C_i(3\dot{u}_i + \frac{\tau}{2}\ddot{u}_i) \tag{17}$$

To obtain the incremental acceleration $\hat{\Delta}\ddot{u}_i$ for the extended interval, the value of $\hat{\Delta}u_i$ obtained from the solution of (15) is substituted into (13). The incremental acceleration $\Delta\ddot{u}_i$ for the normal time interval Δt is then obtained by a simple linear interpolation

$$\Delta\ddot{u} = \hat{\Delta}\ddot{u}/\theta \tag{18}$$

To calculate the incremental velocity $\Delta\dot{u}_i$ and incremental displacement Δu_i corresponding to the normal interval Δt use is made of (11) and (12) with the extended time interval parameter τ substituted for Δt, that is

$$\Delta\dot{u}_i = \ddot{u}_i\Delta t + \frac{1}{2}\Delta\ddot{u}_i\Delta t \tag{19}$$

$$\Delta u_i = \dot{u}_i\Delta t + \frac{1}{2}\ddot{u}_i\Delta t^2 + \frac{1}{6}\Delta\ddot{u}_i\Delta t^2 \tag{20}$$

The displacement u_{i+1} and velocity $\dot{u}_{i+1}$ at the end of the normal time interval are calculated by

$$u_{i+1} = u_i + \Delta u_i \tag{21}$$

$$\dot{u}_{i+1} = \dot{u}_i + \Delta\dot{u}_i \tag{22}$$

The initial acceleration for the next step should be calculated from the condition of dynamic equilibrium at time $t + \Delta t$, thus

$$\ddot{u}_{i+1} = M^{-1}\{P_{i+1} - C_{i+1}\dot{u}_{i+1} - K_{i+1}u_{i+1}\} \tag{23}$$

5 Earthquake Applications

El Centro of 1940 Earthquake Excitation Cases of Study

The excitation data were obtained from the acceleration original record of the first second for El Centro earthquake of 1940 shown in Table 1. And the plot of this record is shown in Figure 7. Note that the ground acceleration is varies with time in units of g, where g is the gravitational acceleration $(g = 386\,in/\sec^2)$. [6]

Table 1: The acceleration record for El Centro earthquake of 1940. [6]

t,sec	$\ddot{u}_g(t), g$	t,sec	$\ddot{u}_g(t), g$	t,sec	$\ddot{u}_g(t), g$
0.000	0.1080	0.429	-0.0237	0.872	-0.0232
0.221	0.0189	0.665	0.0138	0.997	-0.0789
0.374	0.020	0.794	-0.0568	0.161	-0.0001
0.623	0.0094	0.946	-0.0603	0.332	-0.0012
0.789	-0.0387	0.097	0.0159	0.581	0.0425
0.941	-0.0402	0.291	0.0059	0.725	-0.0256
0.042	0.0010	0.471	0.0076	0.877	-0.0343
0.263	0.0001	0.72	-0.0088	1.066	-0.0666

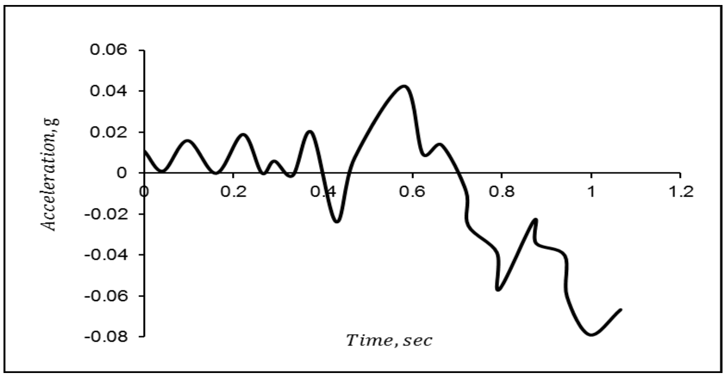

Figure 7: The acceleration record for El Centro earthquake of 1940.

Application 1: Elastic Multi Degree of Freedom System

The analysis of a two-story building system shown in Figure 8. under the effect of the same earthquake. The excitation data were scaled down from the acceleration record for El Centro earthquake of 1940 by a factor of a half, as shown in Table 1. The plastic moment for the columns on the first or second story is $M_P = 154.942\ kip.in$.

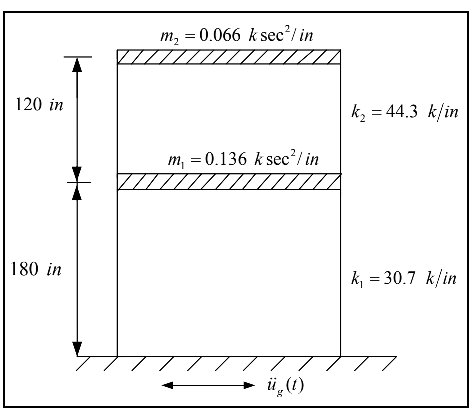

Figure 8: Two-story shear building under earthquake load.

Application 2: Elastoplastic Multi Degree of Freedom System

The analysis of a multi-story building system shown in Figure 8 with elastoplastic behavior. The input data for application (2) is the same as the input data for application (1), but the excitation data for El Centro earthquake of 1940, is used without reduction as listed in Table1.

The Wilson- θ method was used as the method of analysis for applications (1) and (2).
The results for application (1) are plotted in Figures 9 and 10 for stories (1) and (2) respectively. And for application (2) are plotted in Figures 11 and 12 for story 1 and story 2 respectively.

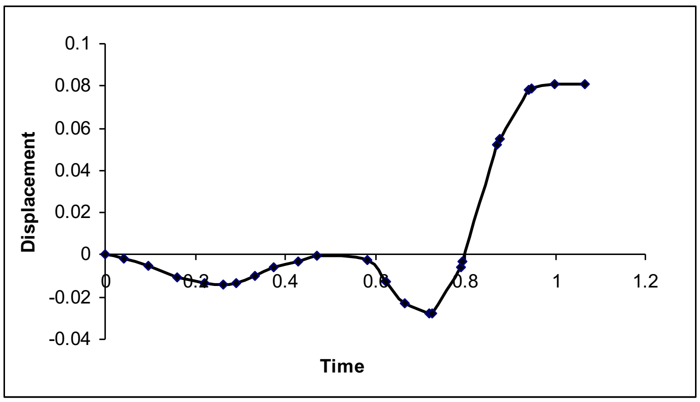

Figure 9: Displacement for earthquake application (1), elastic behavior for Story 1.

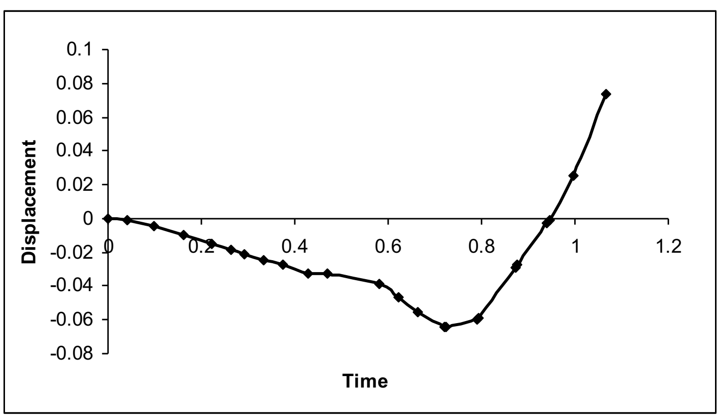

Figure 10: Displacement for earthquake application (1), elastic behavior for Story 2.

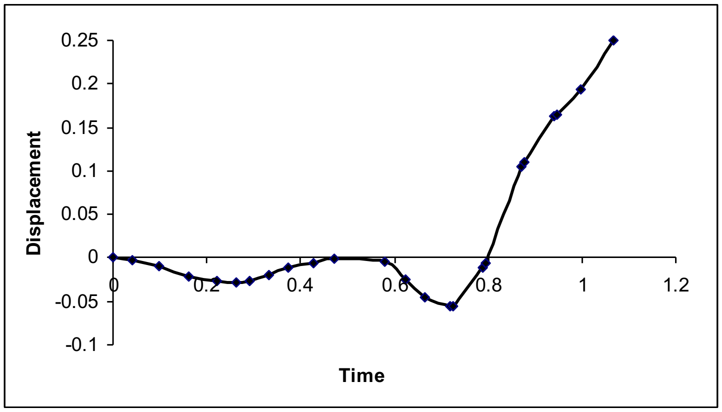

Figure 11: Displacement for earthquake application (2), elastoplastic behavior for Story 1.

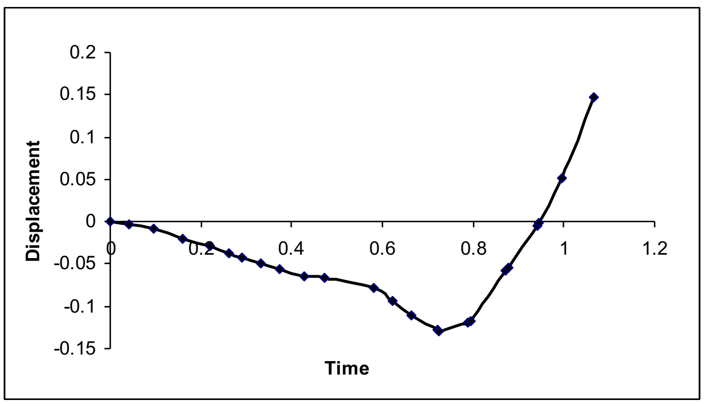

Figure 12: Displacement for earthquake application (2), elastoplastic behavior for Story 2.

Comparison between elastic and elastoplastic displacement response for story (1) and (2) are given in Figures 13 and 14 respectively.

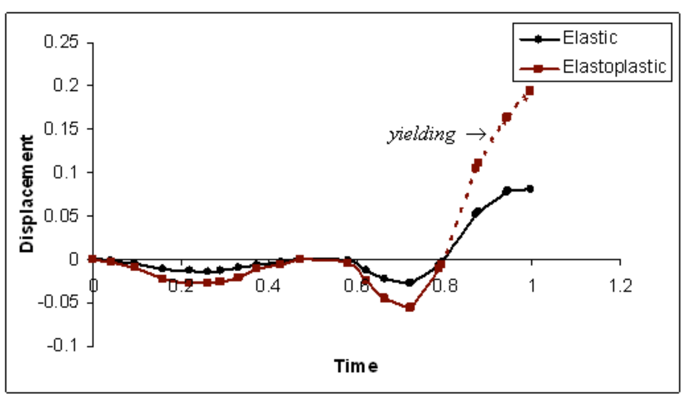

Figure 13: Comparison of elastoplastic behavior with elastic displacement response, story 1 for El Centro earthquake (applications 1 and 2).

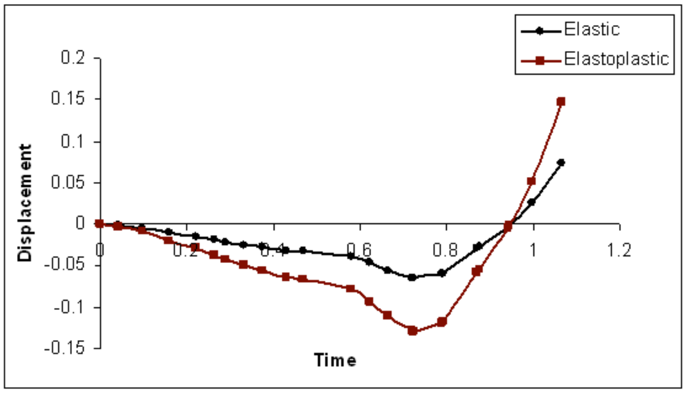

Figure 14: Comparison of elastoplastic behavior with elastic displacement response , story 2 for El Centro earthquake (application 1 and 2).

6 Conclusion

Nonlinear behavior of structures may be due to the inherent nonlinear stress-strain relationship of the material which is called material nonlinearity or due to the changes to geometry (dimensions and configuration) caused by the load, which is called geometrical nonlinearity.

The Wilson-θ method as an unconditionally stable method, serves to assure the numerical stability of the nonlinear solution process regardless of the magnitude selected for the time step. For this reason, Wilson-θ method was chosen as unconditionally stable method for this aspect of nonlinear dynamic analysis. The basic assumption of the Wilson-θ method is that the acceleration varies linearly over the extended interval $\tau = \theta\Delta t$ in which $\theta \geq 1.38$ for unconditional stability.

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