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المجلة ترحب بما يرد عليها من أبحاث وعلى استعداد لنشرها بعد التحكيم .
المجلة تحترم كل الاحترام آراء المحكمين وتعمل بمقتضاها .
كافة الآراء والأفكار المنشورة تعبر عن آراء أصحابها ولا تتحمل المجلة تبعاتها .
يتحمل الباحث مسؤولية الأمانة العلمية وهو المسؤول عما ينشر له .
البحوث المقدمة للنشر لا ترد لأصحابها نشرت أو لم تنشر .

بحوث العدد

- الحركات أبعاض حروف المد واللين .
- التفكير الإيجابي في ضوء بعض المتغيرات الديمغرافية (لدى عينة من الشباب الليبيين)
- أثر التلوث البصري في التأثير على جمالية المدينة "مدينة زليتن كنموذج".
- الاحتجاج بالحديث الضعيف.
- مفهوم الخيال عند سارتر.
- الأَحْكَامُ النَّحْوِيَّةُ المُتَعَلِّقةُ بِالمَوصُولَاتِ الحَرْفِيَّةِ.
- القيم الدلالية للفصل والاعتراض.
- الأبعاد الاجتماعية والثقافية لتنمية ثقافة الحوار في التعليم الجامعي الليبي دراسة ميدانية "جامعة مصراتة أنموذجا".
- العوامل الخمس الكبرى للشخصية وعلاقتها بجنوح الأحداث.
- تقدير الجريان السطحي بحوض وادي جبرون باستخدام نظم المعلومات الجغرافية وتقنيات الاستشعار عن بعد.
- جهود المجامع اللغوية العربية في وضع المصطلحات العلمية.
- استخدام تقنية نظم المعلومات الجغرافية في تحديث الخرائط الورقية (الخرائط الجيولوجية كنموذج).
- ظاهرة القلب الصوتية بين القدامى والمحدثين.
- القول المهم في اعتراض الحصكفي على تعريف ابن هشام للجملة والكلام وأيهما أعم .
- حوادث المرور في ليبيا والأضرار الناجمة عنها.

الافتتاحية

إن الفرد الناجح في حياته، وكذلك المجتمعات والدول هم الذين يحددون أهدافهم، ويضعون في حسبانهم تحقيقها، والوصول إليها، فإذا حدد الفرد والمجتمع لنفسه هدفا فلن يضيع في متاهات الحياة، وسوف يصل إلى المطلوب، فتحديد الهدف أمر مهم ومقوم من مقومات النجاح، لذا على الآباء والمربين، أن يعلموا الأبناء- ومنذ السنوات الأولى في دراستهم- أن يحددوا لأنفسهم أهدافا ينبغي عليهم الاجتهاد من أجل الوصول إليها وتحقيقها، كما يجب أن يعلموهم معايير الأهداف حتى تتوافق مع رغباتهم وقدراتهم.

وعلى المجتمع كله والدول في عالمنا العربي أن يضعوا أهدافا واضحة المعالم للنهوض بالمجتمع يعرفها الصغير قبل الكبير، والجاهل قبل المتعلم، فيسعى الجميع وتتضافر الجهود من أجل تحقيقها وتنفيذها، لا أن تكون طوباوية لا يشعر بها الأفراد، ولا يحسون بقيمتها، فلا يسعون ولا يتعاونون لتحقيقها، بل نجدهم في بعض الأحيان يعملون عكسها لعدم درايتهم بها.

ونتيجة لعدم وجود الأهداف الواضحة المعالم في مجتمعاتنا أفرادا وجماعات لم يصل الفرد منا- عقليا وفكريا واجتماعيا واقتصاديا- إلى مستوى المسؤولية؛ ولم تصل مجتمعاتنا إلى أولى درجات الرقي، فالملاحظ على شبابنا الإهمال والتسيب واللامباة نحو نفسه ونحو مجتمعه، فيقبل بأدنى المراتب ولم يعد في أنظارهم إلا أمرين: المال وبأي وجه كان، والمنصب المرموق دون السعي إلى مؤهلاته، فضعفت لديهم العزيمة، وخارت القوى، ووقع الكثير في سفاسف الأمور.

وفي المقابل نجد أن شبابا كانت أهدافهم واضحة، ومقاصدهم معروفة ارتقوا بفضل ذلك إلى مقامات مرموقة، ووصلوا إلى ما يطمحون إليه، مع شيء التشجيع والمتابعة، فمن سار الطريق وصل.

هيئة التحرير

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Abstract

In this paper, we introduce the concept of fuzzy complex valued metric space by using the notion of complex fuzzy set, moreover, we define the topology induced by this space and some related results of them. In order to illustrate our results we equip the paper with some examples. Also, we state and prove the fuzzy complex valued Banach contraction theorem.

Keywords: Complex valued metric space, fuzzy metric space, complex fuzzy set, fuzzy complex valued metric space, fuzzy complex valued contractive mapping.

1. Introduction

The concept of fuzzy set was first introduced by Zadeh (1965) with his pioneering paper and since then there has been tremendous interest in the subject due to its diverse applications in wide range of scientific areas. In particular, Kramosil and Michalek (1975) introduced the notion of fuzzy metric space and compared this notion with the notion of statistical metric space, that was initially proposed by Schweizer and Sklar (1960). Later on, George and Veeramani (1994) gave a stronger form of metric fuzziness. On the other hand, Ramot et al. (2003) extended the fuzzy set to complex fuzzy set. A complex fuzzy set S is characterized by a membership function $\mu_S(x) = r_S(x)e^{iw_S(x)}$, where $i = \sqrt{-1}$, $r_S(x)$ and $w_S(x)$ are both real valued and $r_S(x) \in [0,1]$, which ranges the interval $[0,1]$ to the unit disc in the plane. Subsequently, Azam et al. (2011) introduced the concept of complex valued metric space, which generalized the classical metric space, and established some fixed point results for mappings satisfying a rational inequality.

In this paper, we introduce the concept of fuzzy complex valued metric space by using the notion of complex fuzzy set, which is a generalization of the corresponding concept of fuzzy metric space that proposed by George and Veeramani (1994) such that we range the interval $[0,1]$ to the unit disc in the plane. Further, we give the topology induced by this space as well as we give some properties about this topology such as Hausdorffness and first countability. Finally, we state and prove the fuzzy complex valued Banach contraction theorem.

2. Preliminaries

In what follows in this section, we recall some notations and definitions that will be utilized in our subsequent discussion.

Definition 2.1. Let $\mathbb{C}$ be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$. Define a partial order $\precsim$ on $\mathbb{C}$ as follows: $z_1 \precsim z_2$ if and only if $Re(z_1) \leq Re(z_2)$; $Im(z_1) \leq Im(z_2)$. Consequently, one can infer that $z_1 \precsim z_2$ if one of the following conditions is satisfied:

(i) $Re(z_1) = Re(z_2), Im(z_1) < Im(z_2)$,

(ii) $Re(z_1) < Re(z_2), Im(z_1) = Im(z_2)$,

(iii) $Re(z_1) < Re(z_2), Im(z_1) < Im(z_2)$,

(iv) $Re(z_1) = Re(z_2), Im(z_1) = Im(z_2)$.

In particular, we write $z_1 \precnsim z_2$, if $z_1 \neq z_2$ and one of (i), (ii), and (iii) is satisfied and we write $z_1 \prec z_2$ if only (iii) is satisfied. Notice that $0 \precsim z_1 \precnsim z_2 \Rightarrow |z_1| < |z_2|$, moreover $z_1 \precsim z_2, z_2 \prec z_3 \Rightarrow z_1 \prec z_3$.

Definition 2.2. Let X be any nonempty set, whereas $\mathbb{C}$ is the set of complex numbers, suppose that the mapping $d : X \times X \longrightarrow \mathbb{C}$, satisfies the following conditions:

1- $0 \precsim d(x,y)$, for all $x, y \in X$ and $d(x,y) = 0$ iff $x = y$,

2- $d(x,y) = d(y,x)$ for all $x, y \in X$,

3- $d(x,y) \precsim d(x,z) + d(z,y)$ for all $x, y, z \in X$.

Then d is called a complex valued metric on X, and (X,d) is called a complex valued metric space.

Definition 2.3. Let (X,d) be a complex valued metric space and (x_n) be a sequence in X and $x \in X$. Then,

i) (x_n) is said to be convergent to x if for every $c \in \mathbb{C}$ with $c \succ 0$ there is $n_0 \in \mathbb{N}$ such that $d(x_n,x) \prec c$ for all $n > n_0$. We denote this by $\lim_{n \to \infty} x_n = x$, or $x_n \longrightarrow x$ as $n \longrightarrow \infty$.

ii) (x_n) is said to be a Cauchy sequence if for every $c \in \mathbb{C}$ with $c \succ 0$ there is $n_0 \in \mathbb{N}$ such that for all $n > n_0, d(x_n, x_{n+m}) \prec c$ where $m \in \mathbb{N}$.

iii) (X,d) is said to be a complete complex valued metric space if every Cauchy sequence is convergent.

Lemma 2.4. Let (X,d) be a complex valued metric space and let (x_n) be a sequence in X. Then

i) (x_n) converges to x if and only if $|d(x_n,x)| \longrightarrow 0$ as $n \longrightarrow \infty$.

ii) (x_n) is a Cauchy sequence if and only if $|d(x_n,x_{n+m})| \longrightarrow 0$ as $n \longrightarrow \infty$,$m \in \mathbb{N}$.

iii) For $c \in \mathbb{C}$ with $c \succ 0$ and $x \in X$, define $B(x,c) = \{y \in X : d(x,y) \prec c\}$ and the family $\beta = \{B(x,c) : x \in X, c \in \mathbb{C}\ with\ c \succ 0\}$, then

$\tau_c = \{A \subset X : \forall x \in A, \exists\ B(x,c) \in \beta, x \in B(x,c) \subset A\}$

is a topology on X.

Definition 2.5. A binary operation $* : [0,1] \times [0,1] \longrightarrow [0,1]$ is a continuous t-norm if $*$ satisfies the following conditions:

1) $*$ is associative and commutative,

2) $*$ is continuous,

3) $a * 1 = a$ for all $a \in [0,1]$,

4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, $a,b,c,d \in [0,1]$.

Definition 2.6. A 3-tuple $(X,M,*)$ is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,\infty)$ satisfying the following conditions:

1) $M(x,y,t) > 0$,

2) $M(x,y,t) = 1$ iff $x = y$,
3) $M(x,y,t) = M(y,x,t)$,
4) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$,
5) $M(x,y,.) : (0,\infty) \longrightarrow [0,1]$ is continuous,
for all $x,y,z \in X$ and $t,s > 0$.

Definition 2.7. Let $(X,M,*)$ be a fuzzy metric space, $x \in X$ and (x_n) be a sequence in X. Then,

i) (x_n) is said to be convergent to x if for any $t > 0$ and any $r \in (0,1)$ there exists a natural number n_0 such that $M(x_n,x,t) > 1-r$ for all $n \geq n_0$. We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$.

ii) (x_n) is said to be a Cauchy sequence if for any $r \in (0,1)$ and any $t > 0$ there exists a natural number n_0 such that $M(x_n,x_m,t) > 1-r$ for all $n,m \geq n_0$.

iii) $(X,M,*)$ is said to be a complete fuzzy metric space if every Cauchy sequence is convergent.

Remark 2.8. Let $(X,M,*)$ be a fuzzy metric space. then

$$\tau = \{A \subset X : x \in A, \text{if and only if}, \exists\ t > 0 \text{ and } r \in (0,1) \text{ such that } B(x,r,t) \subset A\}$$

is a topology on X.

Definition 2.9. A binary operation defined as

$$\circledast : \{a|a \in \mathbb{C}, |a| \leq 1\} \times \{b|b \in \mathbb{C}, |b| \leq 1\} \longrightarrow \{d|d \in \mathbb{C}, |d| \leq 1\}$$

is a continuous phase intersection if $\circledast$ satisfies the following conditions:

1) $\circledast$ is associative and commutative,
2) $\circledast$ is continuous,
3) if $|b| = 1$, then $|a \circledast b| = |a|$ for all $a \in \mathbb{C} : |a| \leq 1$,
4) $|a \circledast b| \leq |c \circledast d|$ whenever $|a| \leq |c|$ and $|b| \leq |d|$, $a,b,c,d \in \{z \in \mathbb{C} : |z| \leq 1\}$.

Remark 2.10. Since Azam et al. (2011) defined a partial order $\precsim$ on the complex numbers $\mathbb{C}$, so that, in the above definition we can replace conditions 3, 4 respectively by the following

3. if $|b| = 1$, then $a \circledast b = a$ for all a in the unit disc of the plane,
4. $a \circledast b \precsim c \circledast d$ whenever $a \precsim c$ and $b \precsim d$ for all a, b, c and d in the unit disc of the plane.

Example 2.11. $a \circledast b = ab$

Example 2.12. $a \circledast b = \min\{a, b\}$

Remark 2.13. For any $r_1 \succ r_2$ we can find r_3 such that $r_1 \circledast r_3 \succsim r_2$ and for any r_4 we can find r_5 such that $r_5 \circledast r_5 \succsim r_4$ whereas r_1, r_2, r_3, r_4 and r_5 are positive complex numbers that belong to the open unit disc in the plane. In addition, for each $r_1 \succsim 0$ and $r_2 \succsim 0$, there exists an element $r \succsim 0$ such that $r \precsim r_1$ and $r \precsim r_2$.

3. Fuzzy complex valued metric spaces

In this section, we define the concept of fuzzy complex valued metric space and the topology induced by this space with some properties.

Definition 3.1. A 3-tuple $(X, M, \circledast)$ is said to be fuzzy complex valued metric space if X is an arbitrary set, $Q = \{c \in \mathbb{C} : c \succ 0\}$, $\circledast$ is a continuous phase intersection and M is a complex fuzzy set on $X^2 \times Q$ satisfying the following conditions:

for all $x, y, z \in \mathrm{X}$ and $t, s, e^{iw} \in Q$ we have

FCVM1) $M(x, y, t) \succ 0$,

FCVM2) $M(x, y, t) = e^{iw}$ iff $x = y$,

FCVM3) $M(x, y, t) = M(y, x, t)$,

FCVM4) $M(x, y, t) \circledast M(y, z, s) \precsim M(x, z, t + s)$,

FCVM5) $M(x, y, \cdot) : Q \to \{c \in \mathbb{C} : |c| \leq 1\}$ is continuous.

Since $\mathbb{R} \subseteq \mathbb{C}$, so that, if we take $Q = [0, \infty)$ and $a \circledast b = ab$ then every fuzzy metric space is a fuzzy complex valued metric space.

Example 3.2. Let $X = \mathbb{R}$, $a \circledast b = ab$ and $M : X^2 \times Q \to \{c \in \mathbb{C} : |c| \leq 1\}$defined by $M(x, y, t) = \left(e^{|x-y|/t}\right)^{-1}$ for all $x, y \in X$, $t \in Q$. Then $(X, M, \circledast)$ is a fuzzy complex valued metric space.

Conditions FCVM1, 2, 3 are obvious. To prove FCVM4 we know that

$$|x-z| \precsim \left(\frac{t+s}{t}\right)|x-y| + \left(\frac{t+s}{s}\right)|y-z|$$

i.e.

$$\frac{|x-z|}{t+s} \precsim \frac{|x-y|}{t} + \frac{|y-z|}{s}$$

Therefore,

$$e^{\frac{|x-z|}{t+s}} \precsim e^{\frac{|x-y|}{t}} e^{\frac{|y-z|}{s}}$$

Thus $M(x,y,t) \circledast M(y,z,s) \precsim M(x,z,t+s)$. Now FCVM5, if we define two mappings n, f as follows $n: Q \to Q$, $n(t) = t$ and $f: Q \to \{c \in \mathbb{C} : |c| \leq 1\}$, $f(s) = \left(e^{|x-y|/s}\right)^{-1}$, then $M(x,y,\cdot): Q \to \{c \in \mathbb{C} : |c| \leq 1\}$ can be considered as composition of n and f. Since n and f are continuous, also M is continuous. Hence $(X, M, \circledast)$ is a fuzzy complex valued metric space.

Remark 3.3. In the above example we can replace $\mathbb{R}$ by any nonempty set X and $|x-y|$ by a metric $d(x,y)$ defined on X. Furthermore, if we replace the phase intersection in the same example by $a \circledast b = min(a,b)$, the example holds.

Example 3.4. Let $X = \mathbb{N}$, $a \circledast b = ab$ and $M: X^2 \times Q \to \{c \in \mathbb{C} : |c| \leq 1\}$ defined by

$$M(x,y,t) = \begin{cases} x/y & if \quad x \leq y \\ y/x & if \quad y \leq x \end{cases}$$

for every $x, y \in X$ and $t \in Q$. It is easily seen that $(X, M, \circledast)$ is a fuzzy complex valued metric space.

Throughout this paper we consider $e^{iw} \in Q$, *where* Q *has mentioned in definition 3.1.*

Lemma 3.5. The mapping $M(x,y,\cdot): Q \to \{c \in \mathbb{C} : |c| \leq 1\}$ is nondecreasing for all $x, y \in X$.

Proof. Suppose that $M(x,y,t) \succ M(x,y,s)$ for $s \succ t \succ 0$whereas $s, t \in Q$. Since $s \succ t$ implies that $s - t \succ 0$. By this supposing and FCVM4 we have

$$M(x,y,t) \circledast M(y,y,s-t) \precsim M(x,y,s) \prec M(x,y,t)$$

$$M(x,y,t) \circledast M(y,y,s-t) \precsim M(x,y,s) \prec M(x,y,t).$$

Since $M(y,y,s-t) = e^{iw}$, so by FCVM2 we obtain that $M(x,y,t) \prec M(x,y,t)$, which is contradiction.

Definition 3.6. Let $(X, M, \circledast)$ be a fuzzy complex valued metric space and $t \in Q$. The open ball $B(x,r,t)$ with center x and radius $r \succ 0$ such that $|r| < 1$ is defined as follows

$$B(x,r,t) = \{y \in X : M(x,y,t) \succ e^{iw} - r\}.$$

Remark 3.7. Let $r, t \in Q$ whereas $|r| < 1$. In a fuzzy complex valued metric space $(X, M, \circledast)$ we can find a complex number t_0, where $0 \prec t_0 \prec t$, such that $M(x,y,t_0) \succ e^{iw} - r$ whenever $M(x,y,t) \succ e^{iw} - r$ for any $x, y \in X$.

Lemma 3.8. Every open ball is an open set.

Proof. Consider an open ball $B(x,r,t)$ and let $y \in B(x,r,t)$, then $M(x,y,t) \succ e^{iw} - r$. So from remark 3.7 we can find $t_0 \in \mathbb{C}$ whereas $0 \prec t_0 \prec t$ such that $M(x,y,t_0) \succ e^{iw} - r$. Suppose that $M(x,y,t_0) = r_0$, since $r_0 \succ e^{iw} - r$, we can find a complex number $s \succ 0$ with $|s| < 1$ such that $r_0 \succ e^{iw} - s \succ e^{iw} - r$. By remark 2.13 we have for a given r_0 and s such that $r_0 \succ e^{iw} - s$ there exists r_1 in Q whereas $|r_1| < 1$ such that $r_0 \circledast r_1 \succsim e^{iw} - s$. Now consider the ball $B(y, e^{iw} - r_1, t - t_0)$ and let $z \in B(y, e^{iw} - r_1, t - t_0)$ implies that

$$M(y,z,t-t_0) \succ r_1.$$

Therefore,

$$M(x,z,t) \succsim M(x,y,t_0) \circledast M(y,z,,t-t_0)$$
$$\succsim r_0 \circledast r_1 \succsim e^{iw} - s \succ e^{iw} - r$$

It is easily seen that $z \in B(x,r,t)$, hence $B(y, e^{iw} - r_1, t - t_0) \subset B(x,r,t)$.

Theorem 3.9. Let $(X, M, \circledast)$ be a fuzzy complex valued metric space. Define

$\tau_{fcv} = \{A \subset X : x \in A \text{ if and only if } \exists\, r,t \in Q \text{ with } |r| < 1 \text{ such that } B(x,r,t) \subseteq A\}$

then τ_{fcv} is a topology on X.

Proof. **1**. Since there is no $x \in \emptyset$ such that $B(x,r,t) \not\subset \emptyset$, thus $\emptyset \in \tau_{fcv}$. Since for any $x \in X$, $r \in Q$; $|r| < 1$ and any $t \in Q$, $B(x,r,t) \subset X$, then $X \in \tau_{fcv}$.

2. Let $A, B \in \tau_{fcv}$ and $x \in A \cap B$, that means $x \in A$ and $x \in B$, so there exist $t_1, t_2 \in Q$, and $r_1, r_2 \in Q$; $|r_1| < 1, |r_2| < 1$ such that $B(x,r_1,t_1) \subset A$, $B(x,r_2,t_2) \subset B$. By remark 2.13 we have that, for any t_1, $t_2 \in Q$ there exists $t \in Q$ such that $t \prec t_1$, $t \prec t_2$. So that, If we take $r = min\{r_1, r_2\}$, then $B(x,r,t) \subset B(x,r_1,t_1) \cap B(x,r_2,t_2) \subset A \cap B$. Thus $A \cap B \in \tau_{fcv}$.

3. Let $A_\alpha \in \tau_{fcv}$ for each $\alpha \in I$ and $x \in \bigcup_{\alpha \in I} A_\alpha$. Then there exists $\alpha_0 \in I$ such that $x \in A_{\alpha_0}$, so there exist t, $r \in Q$; $|r| < 1$ such that $B(x,r,t) \subset A_{\alpha_0}$. Since $A_{\alpha_0} \subset \bigcup_{\alpha \in I} A_\alpha$ implies that $B(x,r,t) \subset \bigcup_{\alpha \in I} A_\alpha$, thus $\bigcup_{\alpha \in I} A_\alpha \in \tau_{fcv}$. Therefore τ_{fcv} is a topology on X.

Theorem 3.10. If $(X, M, \circledast)$ is a fuzzy complex valued metric space, then (X, τ_{fcv}) is Hausdorff.

Proof. Let $x, y \in X$ whereas $x \neq y$. From our definition 3.1 we have $M(x,y,t)$ is a positive complex value lies in the open unit disc, that is $|M(x,y,t)| < 1$, suppose that $M(x,y,t) = r$. From remark 2.13 we have that, for every $r_0 \succ r$ with $|r| < |r_0| < 1$ there exists $r_1 \in Q$; $|r_1| < 1$ such that $r_1 \circledast r_1 \succ r_0$. Now consider the sets $B\left(x, e^{iw} - r_1, \frac{t}{2}\right)$ and $B\left(y, e^{iw} - r_1, \frac{t}{2}\right)$, we have to prove that, these two sets are separated, so we assume that

$$B\left(x, e^{iw} - r_1, \frac{t}{2}\right) \cap B\left(y, e^{iw} - r_1, \frac{t}{2}\right) \neq \emptyset$$

That means, there exists

$$z \in B\left(x, e^{iw} - r_1, \frac{t}{2}\right) \cap B\left(y, e^{iw} - r_1, \frac{t}{2}\right)$$

Then,

$$M\left(x,z,\frac{t}{2}\right) \succ e^{iw} - \left(e^{iw} - r_1\right) = r_1 \text{ and}$$
$$M\left(y,z,\frac{t}{2}\right) \succ e^{iw} - \left(e^{iw} - r_1\right) = r_1$$

However, from FCVM4 we have $M(x,y,t) \succsim M\left(x,z,\frac{t}{2}\right) \circledast M\left(y,z,\frac{t}{2}\right)$ which implies $r \succ r_1 \circledast r_1$, so we obtain that $r \succ r_0 \succ r$, which is contradiction. Hence

$$B\left(x,e^{iw} - r_1,\frac{t}{2}\right) \cap B\left(y,e^{iw} - r_1,\frac{t}{2}\right) = \emptyset.$$

Theorem 3.11. Let $(X,M,\circledast)$ be a fuzzy complex valued metric space. Then (X,τ_{fcv}) is first countable.

Proof. We will prove that $\beta_x = \left\{B\left(x,\frac{1}{n},\frac{t}{n}\right): n \in \mathbb{N}\right\}$ is a local basis for $x \in X$ and $t \in Q$. Let $U \in \tau_{fcv}$ and $x \in U$, since U is open, then there exists $r \succ 0$ which lies in the open unit disc together with $t \in Q$ such that $B(x,r,t) \subset U$. Pick $n \in \mathbb{N}$ such that $\frac{1}{n} \prec r$ and $\frac{t}{n} \prec t$, so we have to show that $B\left(x,\frac{1}{n},\frac{t}{n}\right) \subset B(x,r,t)$. Suppose that $z \in B\left(x,\frac{1}{n},\frac{t}{n}\right)$, then

$$M\left(x,z,\frac{t}{n}\right) \succ e^{iw} - \frac{1}{n} \succ e^{iw} - r.$$

We can see that $\frac{t}{n} \precsim t$, so by lemma 3.5 we have that

$$e^{iw} - r \prec M\left(x,z,\frac{t}{n}\right) \precsim M(x,z,t)$$

which implies $z \in B(x,r,t)$, thus $B\left(x,\frac{1}{n},\frac{t}{n}\right) \subset B(x,r,t) \subset U$. Therefore, β_x is countable local basis for xx. Hence (X,τ_{fcv}) is first countable topological space.

Definition 3.12. Let $(X,M,\circledast)$ be a fuzzy complex valued metric space, $x \in X$ and (x_n) be a sequence in X. Then (x_n) is said to converge to x if for any $t,r \in Q$; $|r| < 1$ there exists $n_0 \in \mathbb{N}$ such that $M(x_n,x,t) \succ e^{iw} - r$ for every $n \geq n_0$. This is denoted by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$.

Theorem 3.13. Let $(X, M, \circledast)$ be a fuzzy complex valued metric space, $x \in X$ and (x_n) be a sequence in X. (x_n) converges to x if and only if $M(x_n, x, t) \to e^{iw}$ as $n \to \infty$, for any $t \in Q$.

Proof. $(\Rightarrow)$ Suppose that $x_n \longrightarrow x$, then there exists a natural number n_0 such that $M(x_n, x, t) \succ e^{iw} - r$ for all $n \geq n_0$, $t, r \in Q$; $|r| < 1$ which implies $e^{iw} - M(x_n, x, t) \prec r$. Thus $M(x_n, x, t) \longrightarrow e^{iw}$ as $n \longrightarrow \infty$.

$(\Leftarrow)$ suppose that $M(x_n, x, t) \longrightarrow e^{iw}$ as $n \longrightarrow \infty$, then there exists a natural number n_0 such that $e^{iw} - M(x_n, x, t) \prec r$ for all $n \geq n_0, t, r \in Q$; $|r| < 1$, so $M(x_n, x, t) \succ e^{iw} - r$. Hence $x_n \longrightarrow x$ as $n \longrightarrow \infty$.

Definition 3.14. Let $(X, M, \circledast)$ be a fuzzy complex valued metric space, $x \in X$ and (x_n) be a sequence in X. Then (x_n) is said to be a Cauchy sequence if for any $t, r \in Q$; $|r| < 1$ there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, t) \succ e^{iw} - r$ for every $n, m \geq n_0$.

Definition 3.15. A fuzzy complex valued metric space is called complete if every Cauchy sequence is convergent.

Definition 3. 16. Let $(X, M, \circledast)$ be a fuzzy complex valued metric space and $A \subset X$. Then A is said to be FCV-bounded if there exists a complex number $r \in Q$ with $|r| < 1$ such that $M(x, y, t) \succ e^{iw} - r$ for all $x, y \in A$ and $t \in Q$.

Theorem 3.17. Every compact subset of a fuzzy complex valued metric space is closed and FCV-bounded.

Proof. Let $(X, M, \circledast)$ be a fuzzy complex valued metric space, A be a compact subset of X and $r, t \in Q$ whereas $|r| < 1$. Since $\{B(x, r, t) : x \in A\}$ is open cover of A, then there exist $x_1, x_2, \dots, x_n \in A$ such that $A \subset \bigcup_{i=1}^{n} B(x_i, r, t)$. For any $x, y \in A$ there exist $1 \leq i, j \leq n$ such that $x \in B(x_i, r, t)$ and $y \in B(x_j, r, t)$, therefore we can write

$$M(x, x_i, t) \succ e^{iw} - r \text{ and } M(x, x_j, t) \succ e^{iw} - r.$$

Take $\alpha = \min \{M(x_i, x_j, t) : 1 \leq i, j \leq n\}$, so we have

$M(x, y, 3t) \succsim M(x, x_i, t) \circledast M(x_i, x_j, t) \circledast M(x_j, y, t)$

$$\succsim \left(e^{iw} - r\right) \circledast \alpha \circledast \left(e^{iw} - r\right)$$

Let $\dot{t} = 3t$ and choose a complex number $s \in Q$ with $|s| < 1$ such that

$$\left(e^{iw} - r\right) \circledast \alpha \circledast \left(e^{iw} - r\right) \succ e^{iw} - s$$

therefore we have $M(x, y, \dot{t}) \succ e^{iw} - s$ for all $x, y \in A$, hence A is FCV-bounded. On the other hand, we know that X is Hausdorff, since every compact subset of a Hausdorff space is closed, then A is closed.

4. Fuzzy complex valued contraction theorem

In this section we will extent the fuzzy Banach contraction theorem, which was given by Gregori and Sapena (2002), for the complete fuzzy complex valued metric space.

Definition 4.1. Let $(X, M, \circledast)$ be a fuzzy complex valued metric space and $f : X \longrightarrow X$ is a self-mapping. Then f is said to be fuzzy complex valued contractive if there exists $k \in (0,1)$ such that

$$\frac{1}{M(f(x), f(y), t)} - e^{iw} \precsim k\left(\frac{1}{M(x, y, t)} - e^{iw}\right)$$

for each $x, y \in X$ and $t \in Q$. k is called the contractive constant of f.

Definition 4.2. Let $(X, M, \circledast)$ be a fuzzy complex valued metric space and (x_n) be a sequence in X. Then (x_n) is said to be fuzzy complex valued contractive if there exists $k \in (0,1)$ such that

$$\frac{1}{M(x_{n+1}, x_{n+2}, t)} - e^{iw} \precsim k\left(\frac{1}{M(x_n, x_{n+1}, t)} - e^{iw}\right)$$

for all $t \in Q, n \in \mathbb{N}$.

Theorem 4.3. (*Fuzzy complex valued Banach contraction theorem*). Let $(X, M, \circledast)$ be a complete fuzzy complex valued metric space in which fuzzy complex valued contractive sequence are Cauchy. Let $T : X \longrightarrow X$ be a fuzzy complex valued contractive mapping with contractive constant k. Then T has a unique fixed point.

Proof. Fix $x \in X$ and let $x_n = T^n(x)$, $n \in \mathbb{N}$. For $t \in Q$, we have

$$\frac{1}{M(T(x),T^2(x),t)} - e^{iw} \precsim k\left(\frac{1}{M(x,x_1,t)} - e^{iw}\right)$$

and by induction

$$\frac{1}{M(x_{n+1},x_{n+2},t)} - e^{iw} \precsim k\left(\frac{1}{M(x_n,x_{n+1},t)} - e^{iw}\right)$$

Then (x_n) is a fuzzy contractive sequence, by assumptions it is a Cauchy sequence and (x_n) converges to y, for some $y \in X$. By theorem 3.13. as $n \longrightarrow \infty$ we have

$$\frac{1}{M(T(y),T(x_n),t)} - e^{iw} \precsim k\left(\frac{1}{M(y,x_n,t)} - e^{iw}\right) \longrightarrow 0$$

Then for each $t \succ 0$ and $\lim_{n\to\infty} M(T(y),T(x_n),t) = e^{iw}$ we have $\lim_{n\to\infty} T(x_n) = T(y)$, that is, $\lim_{n\to\infty} x_{n+1} = T(y)$ and $T(y) = y$.

Now to show the uniqueness assume that $T(z) = z$ for some $z \in X$. For $t \succ 0$, we have

$$\begin{aligned}\frac{1}{M(y,z,t)} - e^{iw} &= \frac{1}{M(T(y),T(z),t)} - e^{iw}\\ &\precsim k\left(\frac{1}{M(y,z,t)} - e^{iw}\right)\\ &= k\left(\frac{1}{M(T(y),T(z),t)} - e^{iw}\right)\\ &\precsim k^2\left(\frac{1}{M(y,z,t)} - e^{iw}\right)\\ &\precsim \cdots \precsim k^n\left(\frac{1}{M(y,z,t)} - e^{iw}\right) \longrightarrow 0\end{aligned}$$

as $n \longrightarrow \infty$. Hence $M(x,y,t) = e^{iw}$ and $y = z$.

Conclusion

in this paper, we defined the notion of fuzzy complex valued metric space which is a generalization of fuzzy metric spaces and then the topology induced by this space. Also, we gave some topological properties, such as Hausdorfness and first countability. After that, the complex valued version of fuzzy Banach contraction theorem was stated and proved. So With the help of these results one can study other fixed point theorems, similar topological properties of this space and problems related to convergence of a sequence.

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