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# مجلة التربوي

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## An Interactive GUESS Method for Solving Nonlinear Constrained Multi-Objective Optimization Problem

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### Abstract

This paper presents an algorithm for tackling nonlinear constrained multi-objective optimization problem. In this algorithm, a GUESS method used to transform a multi-objective problem into a single-objective one. An active set strategy is used together with a multiplier method to transform a single-objective constrained optimization problem into unconstrained one. A line-search globalization strategy is added to the algorithm to solve the obtained unconstrained problem to ensure global convergence.

A Matlab implementation of our algorithm was used in solving one case studies and the result are reported.

**Key Words:** *Multi-objective optimization, GUESS method, Pareto optimal solution, Single objective optimization problem, Line-search, Active-set.*

### 1 Introduction

Nonlinear constrained multi-objective optimization problem (NCMOP) means multiple criteria decision making involving nonlinear functions of decision variables. In these problems, the best possible compromise, that is, Pareto optimal solution, is to be found from an (infinite) number of alternatives represented by decision variables restricted by constraint functions. Thus, enumerating the solutions is impossible. Solving NCMOP usually requires the participation of a human decision maker (DM) who is supposed to have insight into the



problem and who can express preference relations between alternative solutions or objective functions or some other type of preference information.

There are large variety of methods for accomplishing NCMOP, non of them can be said to be generally superior to all of the others. Wang and zhou [19] classi\_ed these methods into four classes according to the participation of the DM in the solution process. If the DM is not involved, we use methods where no articulation of preference of preference information before the solution process. The most extensive class is interactive methods, where the DM speci\_es preference information progressively during the solution process.

The class of interactive method is the most developed one. The interest devoted to this class can be explained by the fact that assuming the DM has enough time and capabilities for co-operation, interactive methods can be presumed to produce the most satisfactory results. Several methods of interactive techniques for multiple criteria decision making have been developed so far (see, [2], [7], [16]).

In this paper, we present one of the simple interactive methods for solving a NMOP called the GUESS method which is used to convert a multi-objective optimization problem (MOP) to a single-objective optimization problem (SOP). This method is an interactive solution method designed to be used with continuous multiple criteria decision problems. It is based on a class of solution methods called reference point methods where by the decision maker generates a sequence of solutions based on a sequence of guesses or aspiration vectors (see, [16]). The method has been compared to several other interactive methods in (see, [1], [6], [8]) and it has performed surprisingly well. The reasons may be its simplicity and flexibility, does not set any specific assumptions on the preference structure of the DM. The DM can change her or his mind since no consistency is required. One can say that decision makers seem to prefer solution methods where they can feel that they are in control.

In this work, we convert the single-objective constrained problem to an unconstrained problem using an active set strategy in [9] combined with the multiplier method. The general idea behind the active-set strategy is to identify at every iteration, the active inequality constraints and treat them as equalities. This allows the use of the well-developed techniques



for solving the equality constrained optimization problems. Many authors have proposed active-set algorithms for solving a general nonlinear programming problems (see, [11], [12], [13], [14]). The main idea of the multiplier methods is to replace the equality constrained optimization problem with a sequence of unconstrained optimization problem and at the same time the penalty parameter needs not to go to infinity (see, [4], [17]).

The line-search strategy used for solving the single-objective constrained and unconstrained optimization problems and multi-objective problems has proved to be very successful both theoretically and practically (see, [13], [14]).

Here, we introduce some notations for subscripted functions denote function values at particular points; for example,  $f_k = f(x_k)$ ,  $\nabla f_k = \nabla f(x_k)$ ,  $L_k = L(x_k, \lambda_k)$ ,  $\nabla_x L_k = \nabla_x L(x_k, \lambda_k)$ , and so on. The matrix  $H_k$  denotes the Hessian of the objective function at the point  $(x_k)$  or an approximation to it. Finally, all norms are  $l_2$ -norms.

The paper is organized as follows. In section 2, mathematical formulation for nonlinear multi-objective optimization problem is described. Section 3, we describe Guess method. In Section 3, we introduce how to solve a single optimization problem with equality and inequality constraints by using an active-set line-search algorithm. Section 4 contains a Matlab implementation of our algorithm. Finally, Section 5 contains concluding remarks.

## 2 Mathematical Formulation for Nonlinear Constrained Multi-Objective Optimization Problem

The general NCMOP is usually written in the form

$$\begin{aligned} & \text{minimize} && [f_1(x), f_2(x), \dots, f_p(x)]^T \\ & \text{subject to} && h(x) = 0, \\ & && g(x) \leq 0. \end{aligned} \quad (1)$$

We assume that  $f_i : \mathfrak{R}^n \rightarrow \mathfrak{R}$  ( $i = 1, 2, \dots, p; p \geq 2$ ),  $h : \mathfrak{R}^n \rightarrow \mathfrak{R}^{me}$ , and  $g : \mathfrak{R}^n \rightarrow \mathfrak{R}^{mi}$  are twice continuously differentiable functions. Let  $y = (y_1, y_2, \dots, y_p)^T$  be the vector of objective functions, i.e.,  $y_i = f_i(x)$  for all  $i = 1, \dots, p$  are objective function values.

To facilitate the presentation, a brief overview of some of the multi-objective optimization



terminology in the form of definitions is given below.

**Definition 1 (Pareto Optimal Solution)**

A feasible decision vector  $x^*$  is called Pareto optimal solution or efficient solution for a NCMOP (1) if there does not exist another feasible decision vector  $x$  such that

- $f_i(x) \leq f_i(x^*)$  for all  $i = 1, 2, \dots, p$ .
- $f_i(x) < f_i(x^*)$  for at least one index  $i$ .

An objective vector  $y^*$  is non-dominated solution if the decision vector corresponding to it is a Pareto optimal solution.[16]

**Definition 2 (Pareto front)**

The set of all non-dominated solutions constitutes the Pareto-optimal set (**Pareto front**).[16]

**Definition 3 (Compromise Solution)**

A compromise solution of the NCMOP (1) is the feasible solution which is preferred by the DM over all other feasible solutions, taking into consideration all criteria contained in the multi-objective functions.[16]

**Definition 4 (Ideal objective vector and nadir objective vector)**

In the Pareto front, an ideal objective vector  $y^a \in \mathfrak{R}^p$  gives lower bounds for the objective functions, and it is obtained by minimizing each objective function individually subject to the constraints, that is, by solving

$$\begin{aligned} & \text{minimize} && f_i(x) \\ & \text{subject to} && h(x) = 0, \\ & && g(x) \leq 0, \end{aligned} \quad (2)$$

for  $i = 1, 2, \dots, p$ .

A nadir objective vector  $y^{nad} \in \mathfrak{R}^p$  giving upper bounds of objective functions values in the Pareto front is usually difficult to calculate, and, thus, its values are usually only approximated by using pay-off tables as follows: Row  $i$  of the payoff table displays the values of all the objective functions calculated at the point where  $f_i$  obtained its minimal



value. The maximal element of the column  $i$  is called  $y_i^{nad}$ . [16]

### 3 GUESS method

The GUESS method is a simple interactive method related to the reference point method. The method is also sometimes called a naive method and it is presented in [Buchanan (1997) [5]]. It is assumed that the ideal and the nadir points are available. The DM specifies a reference point (or a guess)  $\bar{y}^h$  below the nadir point and then the minimum weighted deviation from the nadir point is maximized. Later the DM specifies a new reference point and the iteration continues until the decision makers is satisfied with the solution produced. The DM can also reduce the feasible region by specifying upper and lower bounds at each iteration. Heavy reliance on the availability of the nadir point is the weakness of the GUESS method since the nadir point is not easy to determine and it is usually only an approximation. The scalarized problem to be solved is

$$\begin{aligned} & \text{maximize} \quad \min_{i=1, \dots, P} \left[ \frac{y_i^{nad} - f_i(x)}{y_i^{nad} - \bar{y}_i^h} \right], \\ & \text{subject to} \quad h(x) = 0, \\ & \quad \quad \quad g(x) \leq 0. \end{aligned} \quad (3)$$

Master steps of the GUESS method are presented in the following algorithm

#### Algorithm 1 ( GUESS algorithm )

##### Step 1.

a) Compute  $y_i^{\hat{a}}$  by using Algorithm (2) below to solve problem (2)

for all  $i = 1, \dots, p$ .

b) Compute the nadir objective vector  $y^{nad}$ .

**Step 2.** We present  $y^{\hat{a}}$  and  $y^{nad}$  to the DM. Set  $h = 1$ .

**Step 3.** Ask the DM to specify upper or lower bounds to the objective functions if (s)he so desires. Update the problem, if necessary.

**Step 4.** The DM asked to select a reference point  $\bar{y}_i^h$  for all  $i = 1, \dots, p$  such that



$$y_i^{\hat{a}} \leq \bar{y}_i^h \leq y_i^{nad} .$$

**Step 5.** Compute  $x^h$  by using Algorithm (2) below to solve problems (3).

**Step 6.** Compute the objective vector  $y^h = (f_1(x^h), \dots, f_p(x^h))$ .

**Step 7.** If the DM agree on the objective vector  $y^h$ , then stop.

Else, set  $h = h + 1$  and go to Step 3.

Continuously until the DM is satisfied from the Pareto-optimal set generated.

In **Step 3** we added constraints to problem (3). Thus the scalarized problem(3) becomes

$$\begin{aligned} & \text{maximize} && \min_{i=1, \dots, P} \left[ \frac{y_i^{nad} - f_i(x)}{y_i^{nad} - \bar{y}_i^h} \right], \\ & \text{subject to} && f_i(x) \leq \varepsilon_i \quad \text{for all } i = 1, 2, \dots, p, \\ & && h(x) = 0, \\ & && g(x) \leq 0, \end{aligned} \quad (4)$$

where  $\varepsilon_i$  is upper bounds to the objective functions and it is computed by solving

$$\begin{aligned} & \text{maximize} && f_i(x) \\ & \text{subject to} && h(x) = 0, \\ & && g(x) \leq 0, \end{aligned} \quad (5)$$

for all  $i = 1, \dots, p$ .

The flowchart of the GUESS algorithm 1 as shown in Figure 1.

Using the above algorithm, the constrained multi-objective optimization problem (1) is converted to a constrained single-objective problem (4).

#### 4 Solving Single-Objective Problem

The equality and inequality single-objective constrained problem (4) is converted into an unconstrained problem using an active set strategy together with the multiplier method.

##### 4.1 Active set strategy

The main idea of the active set strategy is to create the active inequality constraints and convert them to equalities at every iteration. Then, by using well developed methods, the equality constrained problem is solved (see, [11], [12], [13], [14]). The main idea in the





penalty method used for solving the equality constrained optimization problem is to replace the problem. The main idea of the multiplier methods is to replace the equality constrained optimization problem with a sequence of unconstrained optimization problem and at the same time the penalty parameter needs not to go to infinity (see, [4], [17]).

The constrained single-objective optimization problem (4) can be formulated as follows:

$$\begin{aligned} & \text{minimize} && F(x) \\ & \text{subject to} && h(x) = 0, \\ & && \tilde{g}(x) \leq 0, \end{aligned} \quad (6)$$

where  $F(x) = \min_{i=1, \dots, p} \left[ \frac{y_i^{nad} - f_i(x)}{y_i^{nad} - \bar{y}_i^h} \right]$  and  $\tilde{g}(x) = [f_i(x) - \varepsilon_i, g(x)]$ .

The functions  $f(x): \mathfrak{R}^n \rightarrow \mathfrak{R}$ ,  $h(x): \mathfrak{R}^n \rightarrow \mathfrak{R}^{me}$ , and  $\tilde{g}(x): \mathfrak{R}^n \rightarrow \mathfrak{R}^{mi+1}$  are twice continuously differentiable.

Following the active set strategy in [9], we define a 0-1 diagonal indicator matrix  $U(x) \in \mathfrak{R}^{mi+1 \times mi+1}$ , whose diagonal entries are

$$u_i(x) = \begin{cases} 1 & \text{if } \tilde{g}_i(x) \geq 0, \\ 0 & \text{if } \tilde{g}_i(x) < 0, \end{cases} \quad (7)$$

Using the above matrix, we transform Problem (6) to the following equality constrained optimization problem

$$\begin{aligned} & \text{minimize} && F(x) \\ & \text{subject to} && h(x) = 0, \\ & && \frac{1}{2} \tilde{g}(x)^T U(x) \tilde{g}(x) = 0. \end{aligned} \quad (8)$$

The above problem can be rewritten as:

$$\begin{aligned} & \text{minimize} && F(x) \\ & \text{subject to} && C(x) = 0, \end{aligned} \quad (9)$$

where  $C(x) \in \mathfrak{R}^{me+1}$  such that  $C(x) = [h(x), \frac{1}{2} \tilde{g}(x)^T U(x) \tilde{g}(x)]$ .



The Lagrangian function associated with the above problem is the function

$$L(x, \lambda) = F(x) + \lambda^T C(x), \quad (10)$$

where  $\lambda \in \mathfrak{R}^{me+1}$  is the Lagrangian multiplier vector associated with the equality constraint  $C(x)$ .

Using an augmented Lagrangian method, we transform the equality constrained optimization problem (9) to the following unconstrained optimization problem

$$\begin{aligned} \text{minimize} \quad & \Phi(x, \lambda; r) = L(x, \lambda) + \frac{r}{2} PC(x) P_2^2, \\ \text{subject to} \quad & x \in \mathfrak{R}^n, \end{aligned} \quad (11)$$

where  $r$  is a parameter usually called the **penalty parameter**.

In the following section, we present a detailed description of the main steps of our line-search algorithm for solving multi-objective problem formulated as unconstrained optimization problem (11).

## 4.2 Active set line-search algorithm outline

In this section, we present the description of the active set line-search algorithm which is used to solve the single-objective problem (11). Then we present the main algorithm that is used to solve the multi-objective optimization problem.

### 4.2.1 Computing a search direction $d_k$

At the point  $x_k$ , we used a quasi-Newton method to find the search direction  $d_k$  which is minimizes the quadratic model

$$m_k(d) = L_k + \nabla_x L_k^T d + \frac{1}{2} d^T B_k d + \frac{r_k}{2} PC_k + \nabla C_k^T d P^2, \quad (12)$$

where  $B_k$  is the Hessian matrix of the Lagrangian function (10) or an approximation to it. If  $(B_k + r_k \nabla C_k \nabla C_k^T)$  is a positive definite matrix, then  $x_k + d_k$  uniquely minimizes the quadratic form (11) where  $d_k$  satisfies

$$(B_k + r_k \nabla C_k \nabla C_k^T) d = -(\nabla_x L_k + r_k \nabla C_k C_k). \quad (13)$$



Since we always require, for all  $k$ , the quasi-Newton direction  $d_k$  be a descent direction, i.e

$$\nabla_x \phi(x_k, \lambda_k; r_k)^T d_k < 0, \quad (14)$$

then the matrix  $(B_k + r_k \nabla C_k \nabla C_k^T)$  must be positive definite (see, [10]). To check the matrix  $(B_k + r_k \nabla C_k \nabla C_k^T)$  is positive definite, we use Tarazaga' s condition (see, [18]).

This condition says that, if

$$\text{trace}(B_k + r_k \nabla C_k \nabla C_k^T) - (n-1)^2 P B_k + r_k \nabla C_k \nabla C_k^T P_F > 0,$$

then  $B_k + r_k \nabla C_k \nabla C_k^T$  is positive definite. Otherwise, we update the diagonal of the matrix  $(B_k + r_k \nabla C_k \nabla C_k^T)$  by adding a positive large number  $\rho$  to the diagonal and compute the search direction  $d_k$  by solving

$$(B_k + r_k \nabla C_k \nabla C_k^T + \rho I) d = -(\nabla_x L_k + r_k \nabla C_k C_k). \quad (15)$$

#### 4.2.2 Computing a step length $\alpha_k$

Once the descent direction  $d_k$  is determined we compute a step length  $\alpha_k$  along the descent direction and set  $x_{k+1} = x_k + \alpha_k d_k$ . We used the backtracking line-search to find the step length  $\alpha_k$ , where  $\alpha_k$  satisfies the sufficient decrease condition

$$\Phi(x_k + \alpha_k d_k, \lambda_k; r_k) \leq \Phi(x_k, \lambda_k; r_k) + \sigma \alpha_k \nabla \Phi(x_k, \lambda_k; r_k)^T d_k, \quad (16)$$

where  $\sigma \in (0,1)$  is a fixed constant. This process is summarized in the following algorithm.

#### Algorithm 2 (Backtracking Line-Search Algorithm)

##### Step 0. (Initialization)

Given  $\sigma \in (0,1)$ ,  $\eta \in (0,1)$ , and set  $\alpha_k = 1$ .

**Step 1.** While  $\Phi(x_k + \alpha_k d_k, \lambda_k; r_k) > \Phi(x_k, \lambda_k; r_k) + \sigma \alpha_k \nabla \Phi(x_k, \lambda_k; r_k)^T d_k$ ,

set  $\alpha_k = \eta \alpha_k$ .

End while.

**Step 2.** Set  $x_{k+1} = x_k + \alpha_k d_k$ .



### 4.2.3 Updating $\lambda_{k+1}$ and $r_k$

Once  $x_{k+1}$  is computed, we update the Lagrange multiplier  $\lambda_k$ . To estimate the Lagrangian multiplier vector  $\lambda_{k+1}$  we solve

$$\underset{\lambda \in \mathbb{R}^{me+1}}{\text{minimize}} \quad P \nabla f_{k+1} + \nabla C_{k+1} \lambda P^2.$$

After updating the Lagrangian multiplier, the penalty parameter is updated. To update  $r_k$ , we use the scheme that was proposed by Bertsekas (1995) [3]. The adjustment scheme is to increase  $r_k$  by multiplication with a factor  $\zeta > 1$  only if the constraint violation as measured by  $PC_{k+1}P$  is not decreased by a factor  $\gamma < 1$  over the previous minimization; i.e.,

$$r^{k+1} = \begin{cases} \zeta r^k & \text{if } PC(x_{k+1})P > \gamma PC(x_k)P, \\ r^k & \text{if } PC(x_{k+1})P \leq \gamma PC(x_k)P, \end{cases} \quad (17)$$

Finally, the algorithm is terminated when  $P \nabla_x L_k P + P \nabla C_k C_k P \leq \varepsilon_1$ , or  $P d_k P \leq \varepsilon_2$ , for some  $\varepsilon_1 > 0$  and  $\varepsilon_2 > 0$ .

### 4.2.4 Active set line-search algorithm

The main steps of our active-set line-search algorithm are explained in detail as follows:

**Algorithm 3** (Active-Set Line-Search Algorithm)

**Step 0.** (Initialization)

Given  $x_0 \in \mathbb{R}^n$ . Compute  $U_0$  and  $\lambda_0$ . Choose  $0 < \sigma < 1$ ,  $0 < \eta < 1$ ,  $\gamma > 0$ ,

$\zeta > 0$ ,  $\varepsilon_1 > 0$ , and  $\varepsilon_2 > 0$ . Set  $r_0 = 1$  and  $k = 0$ .

**Step 1.** If  $P \nabla_x L_k P + P \nabla C_k C_k P \leq \varepsilon_1$ . Then terminate the algorithm.

**Step 2.** Compute the search direction  $d_k$  by solving (13).

**Step 3.** If  $\nabla_x \phi(x_k, \lambda_k; r_k)^T d_k < \varepsilon_1$ , then go to Step 5.

Else, set  $\rho_k = 10^3$ .

While  $\text{trace}(B_k + r_k \nabla C_k \nabla C_k^T + \rho_k I) - (n-1)^{\frac{1}{2}} P B_k + r_k \nabla C_k \nabla C_k^T + \rho_k I P \leq \varepsilon_1$ .



Set  $\rho_k = 2\rho_k$ .

End while.

End if.

**Step 4.** Compute the search direction  $d_k$  by solving (15).

**Step 5.** If  $Pd_k \leq \varepsilon_2$ . Then terminate the algorithm.

**Step 6.** Compute the step length  $\alpha_k$  as follows

a) Set  $\alpha_k = 1$ .

While  $\Phi(x_k + \alpha_k d_k, \lambda_k; r_k) > \Phi(x_k, \lambda_k; r_k) + \sigma \alpha_k \nabla \Phi(x_k, \lambda_k; r_k)^T d_k$ .

Set  $\alpha_k = \eta \alpha_k$ .

End while.

b) Set  $x_{k+1} = x_k + \alpha_k d_k$ .

**Step 7.** Compute  $U_{k+1}$ .

**Step 8.** Compute the Lagrangian multiplier  $\lambda_{k+1}$  by solving

$$\text{minimize}_{\lambda^{me+1} \in \mathbb{R}^n} P \nabla f_{k+1} + \nabla C_{k+1} \lambda P^2.$$

**Step 9.** To update the penalty parameter  $r_k$ .

If  $PC(x_{k+1}) > \gamma PC(x_k)$ ,

then set  $r_{k+1} = \zeta r_k$ .

Else, set  $r_{k+1} = r_k$ .

End if.

**Step 10.** Set  $k = k + 1$  and go to Step 1.

The flowchart of the active-set line-search algorithm 3 as shown in Figure 2.

The main steps of our approach for solving Problem (1) are explained in detail as follows:

#### **Algorithm 4 (Main Algorithm)**

**Step1.** (Transform the multi-objective problem into a single-objective problem)



Run Algorithm (1).

**Step2.** (Construct the payoff table)

**Step3.** (Solve the scalarized problem)

Run algorithm (3) to solve problem (4).

Continuously until the DM is satisfied from the Pareto-optimal set generated.

## 5 Numerical Implementation

In this section, we introduce a multi-objective test problem and we convert this problem to a single objective optimization problem by using GUESS algorithm (1). Then we solve the single objective optimization problem by active set line-search algorithm (3). Our approach used in this study was developed and implemented using MATLAB 7.9 environment with machine epsilon about  $10^{-16}$ .

### 5.1 Implementation details

For implementing the proposed approach, the parameters have been selected as follows:  $\sigma = 0.1, \eta = 0.5, \theta = 2, \gamma = 0.25$ . Successful termination with respect to our line-search algorithm means that the termination condition of the algorithm is met with  $\varepsilon_1 = 10^{-8}$ . On the other hand, unsuccessful termination means that the number of iterations is greater than 500, the number of function evaluations is greater than 800.

### 5.2 Multi-objective test problem

This test problem is an optimal control of continuous Casting by Multi-objective Optimization, presented in [Hwang and Masud, (1979) [15]]. It describes the pollution problems of an artificial river basin. The problem involves a cannery, two cities and a park. Industrial pollution originates from the cannery and the cities are sources of municipal waste. The water quality is measured by dissolved oxygen (DO) concentration. Both the cannery and the two cities reduce currently the waste content by 30 the percent. New treatment facilities are planned. Their costs will reduce the investment return from the cannery and increase the tax rate in the cities. Thus, we have conflicting objectives.

The problem consists of six differentiable objective functions and one nonlinear constraint



function. The three decision variables are the treatment levels of waste discharge at the cannery and in the two cities. The meanings of the objectives are the following. The first three objectives describe the dissolved oxygen (DO) concentration at three different places (in the cities and in the cannery) the fourth objective represents the percent the percent return on investment at the cannery. the addition to the tax rate in the cities is modelled as the fifth and the sixth. The nonlinear constraint restricts the DO level at the end of the river.

The problem formulation is

$$\text{minimize } f_1(x) = -4.75 - 2.27(x_1 - 0.3),$$

$$f_2(x) = -2 - 0.524(x_1 - 0.3) - 2.79(x_2 - 0.3) - 0.882(y_1 - 0.3) - 2.65(y_2 - 0.3),$$

$$f_3(x) = -5.1 - 0.177(x_1 - 0.3) - 0.978(x_2 - 0.3) - 0.216(y_1 - 0.3) - 0.768(y_2 - 0.3),$$

$$f_4(x) = -7.5 + 0.012(59v_1 - 59),$$

$$f_5(x) = 0.0018(532v_2 - 532),$$

$$f_6(x) = 0.0025(450v_3 - 450),$$

$$\text{Subject to } 2.5 - 0.0332(x_1 - 0.3) - 0.0186(x_2 - 0.3) - 3.34(x_3 - 0.3) - 0.0204(y_1 - 0.3)$$

$$-0.778(y_2 - 0.3) - 2.62(y_3 - 0.3) \leq 0,$$

$$0.3 \leq x_i \leq 1, \quad i = 1, 2, 3,$$

$$\text{where } y_i = \frac{0.39}{1.39 - x_i^2}, \quad v_i = \frac{1}{1.09 - x_i^2} \quad \text{and } i = 1, 2, 3. \quad (18)$$

Notice that the first four objective functions are originally to be maximized.

To find the solution of the above nonlinear multi-objective optimization problem.

**First**, Compute the ideal objective vector  $y_i^*$  by solving the following single objective constrained optimization problem using an active set line-search algorithm 3



$$\begin{aligned} & \text{minimize} && f_i(x) \quad i = 1, 2, 3 \\ & \text{subject to} && 2.5 - 0.0332(x_1 - 0.3) - 0.0186(x_2 - 0.3) - 3.34(x_3 - 0.3) \\ & && -0.0204(y_1 - 0.3) - 0.778(y_2 - 0.3) - 2.62(y_3 - 0.3) \leq 0, \\ & && 0.3 \leq x_i \leq 1 \quad , \quad i = 1, 2, 3, \end{aligned} \quad (19)$$

and compute the nadir objective vector  $y^{nad}$  which is approximated from the payoff table.

Set  $h = 1$ .

The values of  $y_i^*$  and  $y^{nad}$  are given in the following Table

$y_i^*$	-6.3407	-6.7922	-6.3240	-7.5002	-0.0004	11.657
$y^{nad}$	-4.7486	-2.3083	-5.1940	-0.21699	9.6824	11.767

**Second**, Compute  $\varepsilon_i$  by solving the following single-objective constrained optimization problem using an active set line-search algorithm 3.

$$\begin{aligned} & \text{maximize} && f_i(x) \quad i = 1, 2, 3 \\ & \text{subject to} && 2.5 - 0.0332(x_1 - 0.3) - 0.0186(x_2 - 0.3) - 3.34(x_3 - 0.3) \\ & && -0.0204(y_1 - 0.3) - 0.778(y_2 - 0.3) - 2.62(y_3 - 0.3) \leq 0, \\ & && 0.3 \leq x_i \leq 1 \quad , \quad i = 1, 2, 3, \end{aligned} \quad (20)$$

The values of  $\varepsilon_i$  is given in the following Table

$\varepsilon_i$	-4.7486	5.7150	-5.1000	7.5004	0.00005	11.530
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**Third**, the DM asked to select a reference point  $\bar{y}_i^h$  for all  $i = 1, \dots, 6$  between the ideal and the nadir objective vectors. Assume that DM select  $\bar{y}_i^h \in R^P$ .

The value of  $\bar{y}_i^h$  given in the following Table

$\bar{y}_i^h$	-5.2251	-4.8321	-5.6413	-5.6624	7.5164	11.6000
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**Fourth,** Compute  $x^h$  by solving the following single-objective constrained optimization

$$\begin{aligned} & \text{maximize} \quad \min_{i=1,\dots,P} \left[ \frac{y_i^{nad} - f_i(x)}{y_i^{nad} - \bar{y}_i^h} \right], \\ & \text{subject to} \quad f_i(x) \leq \varepsilon_i \quad \text{for all } i = 1, 2, \dots, p, \end{aligned} \quad (21)$$

$$\begin{aligned} & 2.5 - 0.0332(x_1 - 0.3) - 0.0186(x_2 - 0.3) - 3.34(x_3 - 0.3) \\ & - 0.0204(y_1 - 0.3) - 0.778(y_2 - 0.3) - 2.62(y_3 - 0.3) \leq 0, \\ & 0.3 \leq x_i \leq 1, \quad i = 1, 2, 3, \end{aligned}$$

where  $y_i = \frac{0.39}{1.39 - x_i^2}$ ,  $v_i = \frac{1}{1.09 - x_i^2}$  and  $i = 1, 2, 3$ .

The numerical results are summarized in the following Table.

$f_i$	-5.3964	-2.2128	-5.1660	-7.2616	0.000006	10.554
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## 6 Concluding Remarks

The proposed algorithm in this paper was presented to solve nonlinear constrained multi-objective optimization problem. The presented algorithm considered as globally search technique to get a pareto-optimal solution. however, we need to select one operating point, which will satisfy the different goals to some extent. Such a solution is called best compromise solution. Consequently, GUESS method can incorporate the DM preference in the optimization process to identify such compromise solution. Also, the method simple to use, does not set any specific assumptions on the preference structure of the DM. The DM can change her or his mind since no consistency is required. One can say that decision makers seem to prefer solution methods where they can feel that they are in control. A numerical example is represented to clarify the proposed algorithm.

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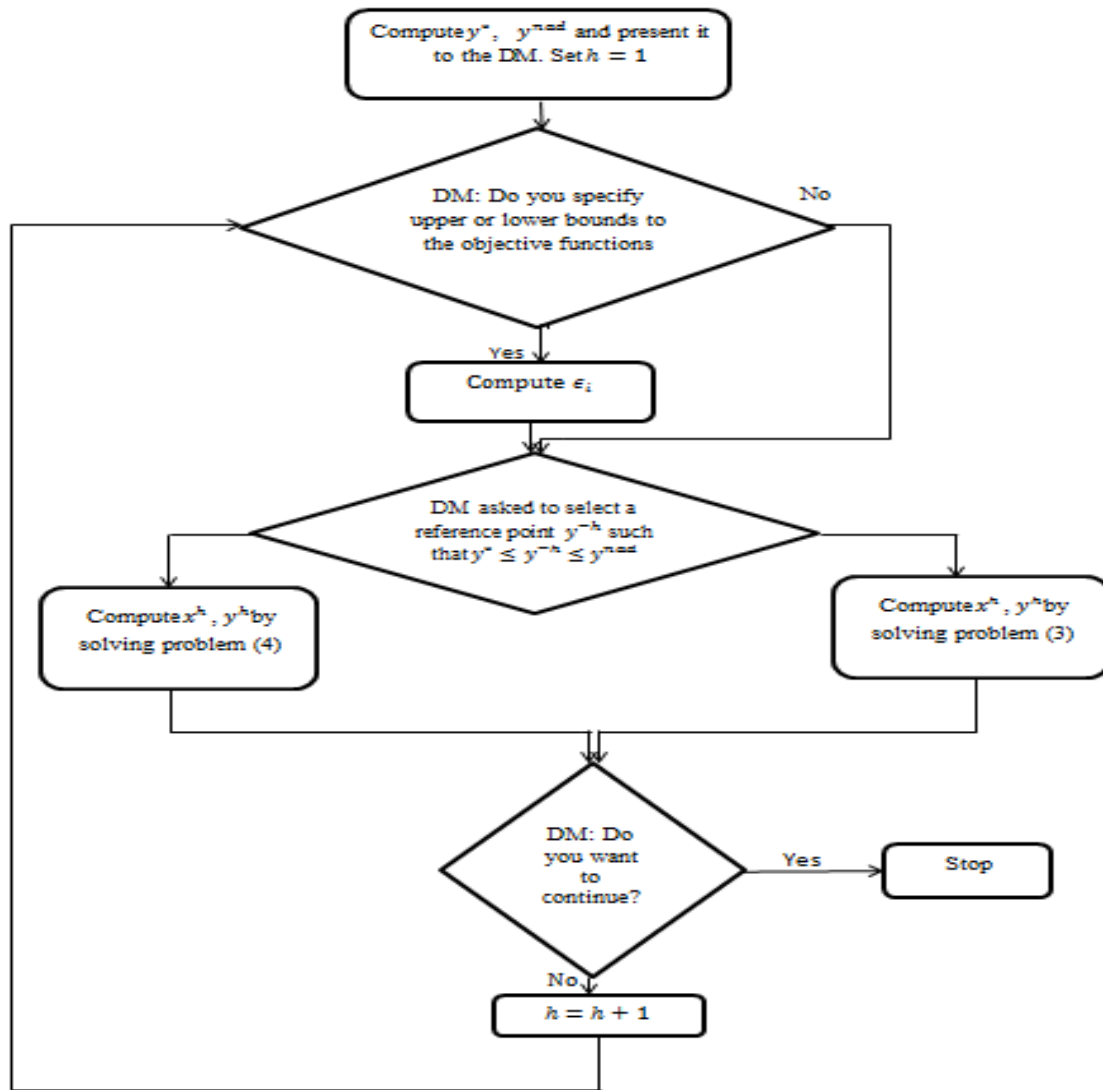


Figure 1: The flowchart of the GUESS algorithm

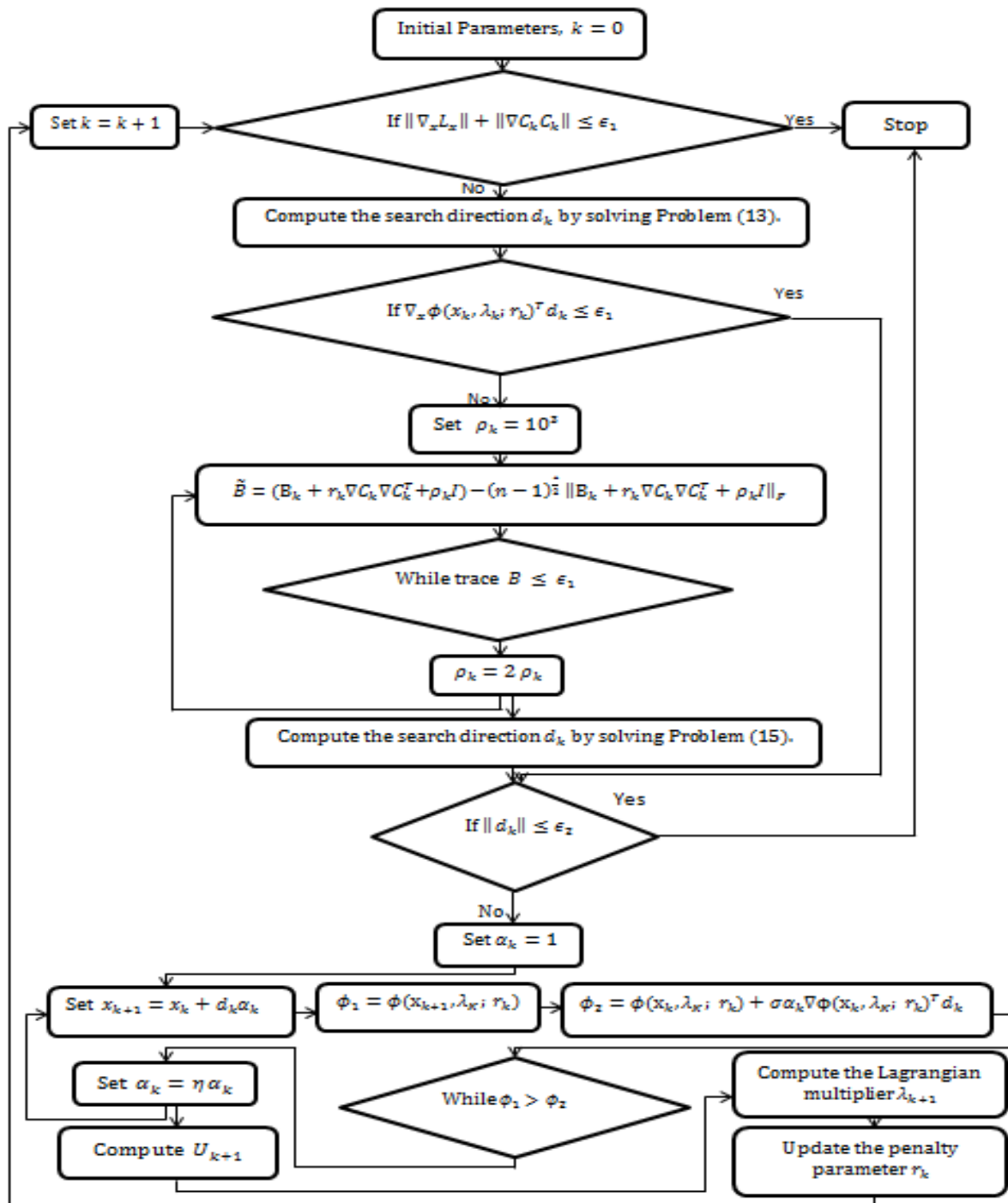


Figure 2: The flowchart of the active-set line-search algorithm



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