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بحوث العدد

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Abstract
The purpose of this paper is to obtain some common fixed point theorems in fuzzy 2-metric space under the condition of occasionally weakly compatible mappings.
Keywords: *Fuzzy 2-metric space, occasionally weakly compatible mappings, coincidence point and common fixed point.*

1. Introduction

It proved a turning point in the development of fuzzy mathematics when the notion of fuzzy set was introduced by Zadeh [8] in 1965. Since then, many authors developed the theory of fuzzy sets and its applications. Following the concept of fuzzy sets, fuzzy metric spaces have been introduced by Kramosil and Michaleck [5] in 1975 which was later modified by George and Veeramani [1] with the help of continuous t-norm in 1994. Consequently in due course of time some metric fixed point results were generalized to fuzzy metric spaces by various authors. Later in 2008, Kumar [17] defined the concept of fuzzy 2-metric space akin to 2-metric space which was introduced by Gähler [12] and obtained a generalization of Banach contraction principle in fuzzy 2-metric spaces. In 1998, Jungck and Rhoades [4] introduced the notion of weakly compatible mappings in metric spaces, after that, Singh and Jain [2] formulated the notion of weakly compatible mappings in fuzzy metric spaces. This condition has further been weakened by introducing the notion of occasionally weakly compatible mappings by Al-Thagafi and Shahzad [10]. While Khan and Sumitra [9] extended the notion of occasionally weakly compatible mappings in fuzzy metric spaces and proved some common fixed point theorems. In recent years, several authors proved various fixed point theorems employing more generalized conditions in difference spaces [2], [3], [6], [7], [11], [13], [14], [15], [16], [18], [19]. In this paper, we prove the existence and uniqueness of some common fixed point theorems for pairs of occasionally weakly compatible mappings in fuzzy 2-metric space by using commutative conditions.

2. Preliminaries

Definition 2.1. Let X be any nonempty set. A fuzzy set M in X is a function with domain X and values in $[0,1]$.

Definition 2.2. A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if it satisfies the following conditions:

(1) $*$ is associative and commutative,

(2) $*$ is continuous function,

(3) $a * 1 = a$ for all $a \in [0,1]$,

(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Examples of t-norm are $a * b = ab$ and $a * b = min\{a, b\}$.

Definition 2.3. The 3-tuple $(X, M,*)$ is called a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set in $X \times X \times [0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$ and $t, s > 0$

(1) $M(x, y, t) > 0$,

(2) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,

(3) $M(x, y, t) = M(y, x, t)$,

(4) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$,

(5) $M(x, y, .): [0, \infty) \rightarrow [0, 1]$ is a left continuous function.

***Note that**, the function value $M(x, y, t)$ can be considered as the degree of nearness between x and y with respect to t.*

Example: Let $X = \mathbb{R}$. Define $a * b = ab$ for all $a, b \in [0,1]$ and

$$M(x, y, t) = \left[exp\frac{|x - y|}{t}\right]^{-1}$$

for all $x, y \in X$ and $t \in (0, \infty)$. Then $(X, M,*)$ is a fuzzy metric space.

Remark: Every metric $d(x, y)$ induces a fuzzy metric $M(x, y, t)$ by the relation $M(x, y, z, t) = \frac{t}{t+d(x,y)}$ such a fuzzy metric is called standard fuzzy metric.

Definition 2.4. Let X be a nonempty set and d be a positive real valued function on $X \times X \times X$ satisfies the following conditions:

(1) For distinct points $x, y \in X$, there exists a point $z \in X$ such that $d(x, y, z) \neq 0$,

(2) $d(x, y, z) = 0$ if at least two of x, y and $z \in X$ are equal,

(3) $d(x, y, z) = d(x, z, y) = d(y, z, x)\ \ \forall\, x, y, z \in X$,

(4) $d(x, y, z) \leq d(x, y, w) + d(x, w, z) + d(w, y, z)\ \ \forall\, x, y, z, w \in X$.

Then the ordered pair (X, d) is called 2-metric space. *Geometrically*, a 2-metric $d(x, y, z)$ represents the area of a triangle with vertices x, y and z in the Euclidean space.

Example: Let $X = \mathbb{R}^3$ and let $d\,(x, y, z)$ the area of the triangle spanned by x, y and z, which may be given explicitly by the formula,

$$d(x, y, z) = |x_1(y_2z_3 - y_3z_2) - x_2(y_1z_3 - y_3z_1) + x_3(y_1z_2 - y_2z_1)|$$

Where $x = (x_1, x_2, x_3),\ y = (y_1, y_2, y_3),\ z = (z_1, z_2, z_3)$. Then (X, d) is a 2-metric space.

Definition 2.5. An operation $*: [0,1] \times [0,1] \times [0,1] \to [0,1]$ is called a continuous t-norm if the following conditions are satisfied: for all $a, b, c, d, e, f \in [0,1]$

(1) $*(a, 1, 1) = a$, $*(0, 0, 0) = 0$,

(2) $*(a, b, c) = *(a, c, b) = *(b, a, c)$,

(3) $*(*(a, b, c), d, e) = *(a, *(b, c, d), e) = *\left(a, b, *(c, d, e)\right)$,

(4) $a * b * c \le d * e * f$ whenever $a \le d, b \le e$ and $c \le f$.

Examples of t-norm are $a * b * c = abc$ and $a * b * c = min\{a, b, c\}$.

Definition 2.6. The 3-tuple $(X, M, *)$ is called a fuzzy 2-metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set in $X \times X \times X \times [0, \infty)$ satisfying the following conditions: for all $x, y, z, w \in X$ and $t_1, t_2, t_3 > 0$

(1) $M(x, y, z, 0) = 0$,

(2) $M(x, y, z, t) = 1$ for all $t > 0$ if and only if at least two of the three points are equal,

(3) $M(x, y, z, t) = M(x, z, y, t) = M(y, z, x, t)$ for all $t > 0$,
(*Symmetry about first three variables*)

(4) $M(x, y, z, t_1 + t_2 + t_3) \ge M(x, y, w, t_1) * M(x, w, z, t_2) * M(w, y, z, t_3)$,
(*This corresponds to tetrahedron inequality in 2-metric space*)

(5) $M(x, y, z, \cdot): [0, \infty) \to [0,1]$ is left continuous .

***Note that**, The function value $M(x, y, z, t)$ may be interpreted as the probability that the area of triangle formed by the three points x, y, z is less than t.*

Example: Let (X, d) be 2-metric space. For all $x, y, z \in X$ and $t > 0$ define

$$M\ (x, y, z, t) = \frac{t}{t + d(x, y, z)}$$

Then $(X, M, *)$ is a fuzzy 2-metric space. Such a fuzzy 2-metric space is known as induced fuzzy 2-metric space.

Lemma 2.7. Let $(X, M, *)$ be a fuzzy 2-metric space. Then $M(x, y, z, .)$ is non-decreasing function for all $x, y, z \in X$.

Definition 2.8. A sequence $\{x_n\}$ in a fuzzy 2-metric space $(X, M, *)$ is said to converge to x in X if and only if $\lim_{n\to\infty} M\ (x_n, x, z, t) = 1 \ \ \forall\, z \in X$ and $t > 0$.

Definition 2.9. Let $(X, M,*)$ be a fuzzy 2-metric space. A sequence $\{x_n\}$ in X is called a Cauchy sequence if and only if $\lim_{n\to\infty} M(x_{n+m}, x_n, z, t) = 1 \ \forall z \in X, m \in \mathbb{N}$, and $t > 0$.

Definition 2.10. A fuzzy 2-metric space $(X, M,*)$ is said to be complete if and only if every Cauchy sequence in X is convergent in X.

Definition 2.11. Let X be a nonempty set. An element $x \in X$ is called a common fixed point of mappings $F: X \to X$ and $T: X \to X$ if $x = T(x) = F(x)$.

Definition 2.12. Let X be a nonempty set. The mappings $F: X \to X$ and $T: X \to X$ are called commutative if $T(F(x)) = F(T(x))$ for all $x \in X$.

Definition 2.13. Let X be a set, F *and* T be self-mappings of X. A point x in X is called a coincidence point of F and T if and only if $F(x) = T(x)$. We shall call $w = F(x) = T(x)$ a point of coincidence of F and T.

Definition 2.14. A pair of mappings F and T is called weakly compatible pair if they commute at coincidence points.

Definition 2.15. Two self-mappings F and T of a set X are occasionally weakly compatible if and only if there is a point x in X that is a coincidence point of F and T at which F and T commute.

Lemma 2.16. Let X be a set, $F$and T be occasionally weakly compatible self-maps of X. If F and T have a unique point of coincidence, $w = F(x) = T(x)$, then w is the unique common fixed point of F and T.

Lemma 2.17. Let $(X, M,*)$ be a fuzzy 2-metric space. If there exists $k \in (0,1)$ such that $M(x, y, z, kt) \geq M(x, y, z, t)$ for all $x, y, z \in X$ and $t > 0$, then $x = y$.

3. Main Results

We have the following theorems.

Theorem 3.1. Let $(X, M,*)$ be a complete fuzzy 2-metric space and let S, T, A, B, P and Q be six self-mappings of X. Let the pairs $\{S, AB\}$ and $\{T, PQ\}$ be occasionally weakly compatible and suppose that

$$AB = BA, AS = SA, BS = SB, TP = PT, PQ = QP \text{ and } TQ = QT.$$

If there exists $k \in (0, 1)$ such that

$$M(Sx, Ty, z, kt) \geq \min\left\{\begin{matrix} M(ABx, PQy, z, t), M(Sx, ABx, z, t), \\ M(Ty, PQy, z, t) \end{matrix}\right\} \qquad (1)$$

for all $x, y, z \in X$ and for all $t > 0$, then there is a unique common fixed point of S, T, A, B, P and Q.

Proof. Since the pairs $\{S, AB\}$ and $\{T, PQ\}$ are occasionally weakly compatible, so there are points $x, y \in X$ such that

$$Sx = ABx;\ SABx = ABSx \ \text{ and } \ Ty = PQy; TPQy = PQTy.$$

We claim that $Sx = Ty$. If $Sx \neq Ty$, then there exists a positive real number t such that $M(Sx, Ty, z, t) < 1$. By inequality (1) we obtain

$$M(Sx, Ty, z, kt) \geq \min\begin{Bmatrix} M(Sx, Ty, z, t), M(Sx, Sx, z, t), \\ M(Ty, Ty, z, t) \end{Bmatrix}$$
$$= \min\{M(Sx, Ty, z, t), 1,1\}$$
$$= M(Sx, Ty, z, t).$$

Therefore $Sx = Ty$, so we have $Sx = ABx = Ty = PQy$. Suppose that there is another point u such that $Su = ABu$ then by inequality (1) we have $Su = ABu = Ty = PQy$, so $Sx = Su = w$ and $w = Sx = ABx$ is the unique point of coincidence of S and AB. By Lemma 2.16 w is the unique common fixed point of S and AB. Similarly, there is a unique point $r \in X$ such that $r = Tr = PQr$.

Now assume that $w \neq$ r. So we have

$$M(w, r, z, kt) = M(Sw, Tr, z, kt)$$

$$\geq \min\{M(ABw, PQr, z, t), M(Sw, ABw, z, t), M(Tr, PQr, z, t)\}$$
$$= \min\{M(w, r, z, t), M(w, w, z, t), M(r, r, z, t)\}$$
$$= \min\{M(w, r, z, t), 1,1\} = M(w, r, z, t).$$

Therefore we have $r = w$, by Lemma 2.16 w is a common fixed point of S, T, AB and PQ.

Putting $x = Aw$ and $y = w$ in inequality (1) we get

$$M(SAw, Tw, z, kt) \geq \min\begin{Bmatrix} M(ABAw, PQw, z, t), M(SAw, ABAw, z, t), \\ M(Tw, PQw, z, t) \end{Bmatrix}$$
$$M(ASw, Tw, z, kt) \geq \min\begin{Bmatrix} M(AABw, PQw, z, t), M(ASw, AABw, z, t), \\ M(Tw, PQw, z, t) \end{Bmatrix}$$
$$M(Aw, w, z, kt) \geq \min\begin{Bmatrix} M(Aw, w, z, t), M(Aw, Aw, z, t), \\ M(w, w, z, t) \end{Bmatrix}$$
$$= \min\{M(Aw, w, z, t), 1,1\} = M(Aw, w, z, t)$$

implies that $Aw = w$. Next, put $x = Bw$ and $y = w$ we obtain

$$M(SBw, Tw, z, kt) \geq \min\begin{Bmatrix} M(ABBw, PQw, z, t), M(SBw, ABBw, z, t), \\ M(Tw, PQw, z, t) \end{Bmatrix}$$
$$M(BSw, Tw, z, kt) \geq \min\begin{Bmatrix} M(BABw, PQw, z, t), M(BSw, BABw, z, t), \\ M(Tw, PQw, z, t) \end{Bmatrix}$$

$$M(Sw,w,z,kt) \geq \min\left\{\begin{matrix} M(Bw,w,z,t), M(Bw,Bw,z,t), \\ M(w,w,z,t) \end{matrix}\right\}$$
$$= \min\{M(Bw,w,z,t),1,1\} = M(Bw,w,z,t)$$

therefore $Bw = w$. By putting $x = w$ and $y = Pw$ we get

$$M(Sw,TPw,z,kt) \geq \min\left\{\begin{matrix} M(ABw,PQPw,z,t), M(Sw,ABw,z,t), \\ M(TPw,PQPw,z,t) \end{matrix}\right\}$$
$$M(Sw,PTw,z,kt) \geq \min\left\{\begin{matrix} M(ABw,PPQw,z,t), M(Sw,ABw,z,t), \\ M(PTw,PPQw,z,t) \end{matrix}\right\}$$
$$M(w,Pw,z,kt) \geq \min\left\{\begin{matrix} M(w,Pw,z,t), M(w,w,z,t), \\ M(Pw,Pw,z,t) \end{matrix}\right\}$$
$$= \min\{M(w,Pw,z,t),1,1\} = M(w,Pw,z,t)$$

thus $Pw = w$. Next, put $x = w$ and $y = Qw$ we have

$$M(Sw,TQw,z,kt) \geq \min\left\{\begin{matrix} M(ABw,PQQw,z,t), M(Sw,ABw,z,t), \\ M(TQw,PQQw,z,t) \end{matrix}\right\}$$
$$M(Sw,QTw,z,kt) \geq \min\left\{\begin{matrix} M(ABw,QPQw,z,t), M(Sw,ABw,z,t), \\ M(QTw,QPQw,z,t) \end{matrix}\right\}$$
$$M(w,Qw,z,kt) \geq \min\left\{\begin{matrix} M(w,Qw,z,t), M(w,w,z,t), \\ M(Qw,Qw,z,t) \end{matrix}\right\}$$
$$= \min\{M(w,Qw,z,t),1,1\} = M(w,Qw,z,t)$$

hence $Qw = w$. From the previse procedure we have

$$Sw = Tw = Aw = Bw = Pw = Qw = w.$$

Therefore, w is a common fixed point of S, T, A, B, P and Q. The uniqueness of the common fixed point holds from inequality (1).

Theorem 3.2. Let $(X, M, *)$ be a complete fuzzy 2-metric space and let S, T, A, B, P and Q be six self-mappings of X. Let the pairs $\{S, AB\}$ and $\{T, PQ\}$ be occasionally weakly compatible and suppose that

$$AB = BA, AS = SA, BS = SB, TP = PT, PQ = QP \text{ and } TQ = QT.$$

If there exists $k \in (0,1)$ such that

$$M(Sx,Ty,z,kt) \geq \varphi\left(min\left\{\begin{matrix} M(ABx,PQy,z,t), M(Sx,ABx,z,t), \\ M(Ty,PQy,z,t) \end{matrix}\right\}\right) \qquad (2)$$

for all $x, y, z \in X, t > 0$ and $\varphi: [0,1] \to [0,1]$ such that $\varphi(h) > h$ for all $0 < h < 1$, then there exists a unique common fixed point of S, T, A, B, P and Q .

Proof. The proof follows from Theorem 3.1.

Theorem 3.3. Let $(X, M, *)$ be a complete fuzzy 2-metric space and let S, T, A, B, P and Q be six self-mappings of X. Let the pairs $\{S, AB\}$ and $\{T, PQ\}$ be occasionally weakly compatible and suppose that

$$AB = BA, AS = SA, BS = SB, TP = PT, PQ = QP \text{ and } TQ = QT.$$

If there exists $k \in (0,1)$ such that

$$M(Sx,Ty,z,kt) \geq \varphi\big(M(ABx,PQy,z,t), M(Sx,ABx,z,t), M(Ty,PQy,z,t)\big) \qquad (3)$$

for all $x,y,z \in X, t > 0$ and $\varphi: [0,1] \times [0,1] \times [0,1] \to [0,1]$ such that $\varphi(h,1,1) > h$ for all $0 < h < 1$, then there exists a unique common fixed point of S,T,A,B,P and Q.

Proof. Since the pairs $\{S,AB\}$ and $\{T,PQ\}$ are occasionally weakly compatible, there are points $x,y \in X$ such that $Sx = ABx$ and $Ty = PQy$. We claim that $Sx = Ty$. By inequality (3) we have

$$\begin{aligned} M(Sx,Ty,z,kt) &\geq \varphi(M(ABx,PQy,z,t), M(Sx,ABx,z,t), M(Ty,PQy,z,t)) \\ &= \varphi(M(Sx,Ty,z,t), M(Sx,Sx,z,t), M(Ty,Ty,z,t)) \\ &= \varphi(M(Sx,Ty,z,t),1,1) \\ &> M(Sx,Ty,z,t) \end{aligned}$$

Therefore $Sx = Ty$, so we have $Sx = ABx = Ty = PQy$. Suppose that there is another point $u \in X$ such that $Su = ABu$ then by inequality (3) we have $Su = ABu = Ty = PQy$, so $Sx = Su = w$ and $w = Sx = ABx$ is the unique point of coincidence of S and AB. By Lemma 2.16 w is the unique common fixed point of S and AB. Similarly, there is a unique point $r \in X$ such that $r = Tr = PQr$.

Now assume that $w \neq$ r. So we have

$$\begin{aligned} M(w,r,z,kt) &= M(Sw,Tr,z,kt) \\ &\geq \varphi\big(M(ABw,PQr,z,t), M(Sw,ABw,z,t), M(Tr,PQr,z,t)\big) \\ &= \varphi\big(M(w,r,z,t), M(w,w,z,t), M(r,r,z,t)\big) \\ &= \varphi(M(w,r,z,t),1,1) > M(w,r,z,t) \end{aligned}$$

Therefore we have $r = w$, by Lemma 2.16 w is a common fixed point of S,T,AB and PQ.

Putting $x = Aw$ and $y = w$ in inequality (3) we get

$$M(SAw,Tw,z,kt) \geq \varphi\big(M(ABAw,PQw,z,t), M(SAw,ABAw,z,t), M(Tw,PQw,z,t)\big)$$

$$M(ASw,Tw,z,kt) \geq \varphi\big(M(AABw,PQw,z,t), M(ASw,AABw,z,t), M(Tw,PQw,z,t)\big)$$

$$\begin{aligned} M(Aw,w,z,kt) &\geq \varphi\big(M(Aw,w,z,t), M(Aw,Aw,z,t), M(w,w,z,t)\big) \\ &= \varphi(M(Aw,w,z,t),1,1) > M(Aw,w,z,t) \end{aligned}$$

implies that $Aw = w$. Next, put $x = Bw$ and $y = w$ we obtain

$$M(SBw,Tw,z,kt) \geq \varphi\big(M(ABBw,PQw,z,t),M(SBw,ABBw,z,t),M(Tw,PQw,z,t)\big)$$

$$M(BSw,Tw,z,kt) \geq \varphi\big(M(BABw,PQw,z,t),M(BSw,BABw,z,t),M(Tw,PQw,z,t)\big)$$

$$M(Sw,w,z,kt) \geq \varphi\big(M(Bw,w,z,t),M(Bw,Bw,z,t),M(w,w,z,t)\big) = \varphi(M(Bw,w,z,t),1,1) > M(Bw,w,z,t)$$

therefore $Bw = w$. By putting $x = w$ and $y = Pw$ we get

$$M(Sw,TPw,z,kt) \geq \varphi\big(M(ABw,PQPw,z,t),M(Sw,ABw,z,t),M(TPw,PQPw,z,t)\big)$$

$$M(Sw,PTw,z,kt) \geq \varphi\big(M(ABw,PPQw,z,t),M(Sw,ABw,z,t),M(PTw,PPQw,z,t)\big)$$

$$M(w,Pw,z,kt) \geq \varphi\big(M(w,Pw,z,t),M(w,w,z,t),M(Pw,Pw,z,t)\big) = \varphi(M(w,Pw,z,t),1,1) > M(w,Pw,z,t)$$

thus $Pw = w$. Next, put $x = w$ and $y = Qw$ we have

$$M(Sw,TQw,z,kt) \geq \varphi\big(M(ABw,PQQw,z,t),M(Sw,ABw,z,t),M(TQw,PQQw,z,t)\big)$$

$$M(Sw,QTw,z,kt) \geq \varphi\big(M(ABw,QPQw,z,t),M(Sw,ABw,z,t),M(QTw,QPQw,z,t)\big)$$

$$M(w,Qw,z,kt) \geq \varphi\big(M(w,Qw,z,t),M(w,w,z,t),M(Qw,Qw,z,t)\big) = \varphi(M(w,Qw,z,t),1,1) > M(w,Qw,z,t)$$

hence $Qw = w$. From the previse procedure we have

$$Sw = Tw = Aw = Bw = Pw = Qw = w.$$

Therefore, w is a common fixed point of S,T,A,B,P and Q. The uniqueness of the common fixed point holds from inequality (3).

Theorem 3.4. Let $(X,M,*)$ be a complete fuzzy 2-metric space and let S,A and B be three self-mappings of X. Let the pair $\{S,AB\}$ be occasionally weakly compatible and suppose that $AB = BA, AS = SA$ and $BS = SB$ If there exists a point $k \in (0,1)$ such that

$$M(Sx,Sy,z,kt) \geq \alpha M(ABx,ABy,z,t) + \beta \min \begin{Bmatrix} M(ABx,ABy,z,t), \\ M(Sx,ABx,z,t), \\ M(Sy,ABy,z,t) \end{Bmatrix} \qquad (4)$$

for all $x, y, z \in X$ and $t > 0$, where $\alpha, \beta > 0, \alpha + \beta > 1$. Then S, A and B have a unique common fixed point.

Proof. Since the pair $\{S, AB\}$ is occasionally weakly compatible, so there is a point $x \in X$ such that $Sx = ABx$. Suppose that there exist another point $y \in X$ for which $Sy = ABy$. We claim that $Sx = Sy$. If $Sx \neq Sy$ then by inequality (4) we have

$$M(Sx, Sy, z, kt) \geq \alpha M(ABx, ABy, z, t) + \beta \min \begin{Bmatrix} M(ABx, ABy, z, t), \\ M(Sx, ABx, z, t), \\ M(Sy, ABy, z, t) \end{Bmatrix}$$

$$= \alpha M(Sx, Sy, z, t) + \beta \min \begin{Bmatrix} M(Sx, Sy, z, t), \\ M(Sx, Sx, z, t), \\ M(Sy, Sy, z, t) \end{Bmatrix}$$

$$= (\ \alpha M(Sx, Sy, z, t) + \beta \min\{M(Sx, Sy, z, t), 1, 1\}\ )$$

$$= (\alpha + \beta) M(Sx, Sy, z, t)$$

which is contradiction, since $(\alpha + \beta) > 1$, therefore $Sx = Sy$, so $Sx = ABx = Sy = ABy$. Suppose that there is another point $u \in X$ such that $Su = Sy$ then by inequality (4) we have $Su = ABu = Sy = ABy$, so $Sx = Su = w$ and $w = Sx = ABx$ is the unique point of coincidence of S and AB. By Lemma 2.16, S and AB have a unique fixed point, which is w.

Putting $x = Aw$ and $y = w$ in inequality (4) we get

$$M(SAw, Sw, z, kt)$$

$$\geq \alpha M(ABAw, ABw, z, t) + \beta \min \begin{Bmatrix} M(ABAw, ABw, z, t), \\ M(SAw, ABAw, z, t), \\ M(Sw, ABw, z, t) \end{Bmatrix}$$

$$M(ASw, Sw, z, kt)$$

$$\geq \alpha M(AABw, ABw, z, t) + \beta \min \begin{Bmatrix} M(AABw, ABw, z, t), \\ M(ASw, AABw, z, t), \\ M(Sw, ABw, z, t) \end{Bmatrix}$$

$$M(Aw, w, z, kt) \geq \alpha M(Aw, w, z, t) + \beta \min \begin{Bmatrix} M(Aw, w, z, t), \\ M(Aw, Aw, z, t), \\ M(w, w, z, t) \end{Bmatrix}$$

$$= \alpha M(Aw, w, z, t) + \beta \min\{M(Aw, w, z, t), 1,1\}$$

$$= (\alpha + \beta) M(Aw, w, z, t)$$

implies that $Aw = w$. Now, put $x = Bw$ and $y = w$ we obtain

$$M(SBw,Sw,z,kt) \geq \alpha M(ABBw,ABw,z,t)+\beta \min\begin{Bmatrix} M(ABBw,ABw,z,t), \\ M(SBw,ABBw,z,t), \\ M(Sw,ABw,z,t)\end{Bmatrix}$$

$$M(BSw,Sw,z,kt) \geq \alpha M(BABw,ABw,z,t)+\beta \min\begin{Bmatrix} M(BABw,ABw,z,t), \\ M(BSw,BABw,z,t), \\ M(Sw,ABw,z,t)\end{Bmatrix}$$

$$M(Bw,w,z,kt) \geq \alpha M(Bw,w,z,t)+\beta \min\begin{Bmatrix} M(Bw,w,z,t), \\ M(Bw,Bw,z,t), \\ M(w,w,z,t)\end{Bmatrix}$$
$$= \alpha M(Bw,w,z,t)+\beta \min\{M(Bw,w,z,t),1,1\}$$
$$= (\alpha+\beta)M(Bw,w,z,t)$$

Thus $Bw = w$. From the previse procedure we have

$$Sw = Aw = Bw = w.$$

Therefore, w is a common fixed point of S, A and B. The uniqueness of the common fixed point holds from inequality (4).

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الفهـــــــــــرس

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